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Measuring Solution Quality of Multiobjective Evolutionary Algorithms

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Abstract. In single objective optimization problems it is easy to find a metric that allows different solutions to be compared and ranked even if the optimum is not known. In a multiobjective optimization (MOO), however, a Pareto front must be considered rather than a single optimal point. A large number of methods for solving MOO problems have been developed. To compare these methods rigorously, or to measure the performance of a particular MOO algorithm quantitatively, a variety of performance metrics have been proposed. This paper presents a new performance metric based on Ideal and nadir points that should enable a designer to either monitor the quality of an observed Pareto solution set as obtained by a multiobjective optimization method, or compare the quality of observed Pareto solution sets as reported by different multiobjective optimization methods, also measuring solution quality are useful during execution of a heuristic procedure, namely as stopping rules. Numerical analysis is used to demonstrate the calculation of this metric for an observed Pareto solution set.. The results clearly show that our performance metric gives a quick and good means of assessing progress towards true Pareto optimal solution.

Keywords: Multiobjective Optimization; Performance Metric; Genetic Algorithms, Ideal Point , Nadir Point.

1. Introduction

The field of multiple-objective linear programming (MOLP) has attracted a lot of attention since the early 1970s and many approaches were developed to address these problems. MOLP problems do not generally have a unique solution like in single-objective linear programming. Instead, a family of "reasonable" (nondominated) solutions is identified[1,5,9], and the interaction of a DM is required to find the "most preferred" one. It is important for a designer to know how good an observed Pareto solution set is that multiobjective optimization method attains. Indeed, knowledge of the goodness of observed Pareto solution set should enable the designer monitor and potentially improve the performance of a multiobjective optimization method.

In the last 10 years, many new multiobjective optimization methods have been created, most of them built on well known metaheuristics like simulated annealing or evolutionary algorithms[4,3,7,8]. The main goal of multiobjective optimization is to find a set of parameters (a solution) so as to optimize concurrently some objective functions. Earlier multiobjective evolutionary algorithms (MOEAs) paid emphasis on getting more and more close to the true Pareto optimal (PO) front in the objective space. Comparing the performance of different MOEAs is complicated by the fact that the result of a MOEAs run is not a single scalar value but a vector of objective values. Also as we know that there are two distinct goals (Fig. 1) in multiobjective optimization[4],

(i) Discover solutions as close to the PO solutions as possible (which requires search towards the PO front).

(ii) Find solutions as diverse as possible in the obtained nondominated front (which requires search along the PO front).

In multiobjective evolutionary algorithms, various stopping criteria have been developed. One, a total number of iterations, If a pre-defined maximum generation number is reached then the algorithm stop. Second, stopping criterion is based on the nature of GA. In GA, convergence may occur when all bit positions in all strings are identical. All these sopping criteria does not guarantee convergence of the algorithm.

Also, the issue of evaluating approximations for Pareto set is addressed. Such evaluations are useful when performing experimental comparisons of different multiple objective heuristic algorithms, or when defining stopping rules of multiple objective heuristic algorithms

Our aim in this paper is to propose a new metric that allows different solutions to be compared and ranked even if the optimum is not known. Also it enable a DM to either monitor the quality of an observed Pareto solution set as obtained by a multiobjective optimization methods which help him to take a decision to stop the algorithm, or compare the quality of observed Pareto solution sets as reported by different multiobjective optimization.

This paper is organized as follows; some of more recent and important metrics of performance are reviewed in section 2. Section 3 gives out new metric for convergence. The numerical analysis is discussed in section 4. Conclusion follows in section 6.

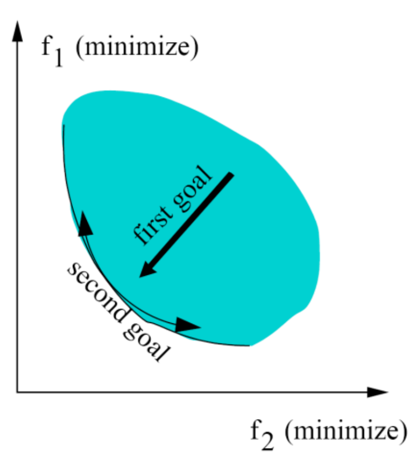


Fig. (1). Two goals of multiobjective optimization.

2. Performance Metrics: A Review

According to Deb [4] the existing performance metrics can be classified into three classes: metrics for convergence, metrics for diversity and metrics for both convergence and diversity. For more details the reader is referred to [4,2,10]

Many metrics for measuring the convergence of a set of nondominated solutions towards the Pareto front have been proposed. Almost all of these metrics were constructed in order to directly compare two sets of nondominated solutions. There are also approaches which compare a set of nondominated solutions with a set of Pareto optimal solutions if the true Pareto front is known. In what follows we review some existing metrics for convergence.

(2.1) The S metric

The S metric has been introduced by Zitzler in [11] and improved in [12]. The S metric measures how much of the objective space is dominated by a given nondominated set A.

Consider a nondominated minimization solution set: $A = \{z_1, z_2, z_3\}$ in a normalized design objective space (with the upper bound of the feasible region shifted to the point z^{ref}). The size of the dominated space by set A, denoted by S(A), is defined as the volume of the union of hypercubes $\{C_1, C_2, .. C_t\}$ where C_i is a hypercube whose two opposite vertices are z_i and z^{ref} of the objective space. Fig. (2), for instance, shows the three hypercubes generated by the non-dominated set $A = \{z_1, z_2, z_3\}$ The volume of the union of these two hypercubes measures S(A).

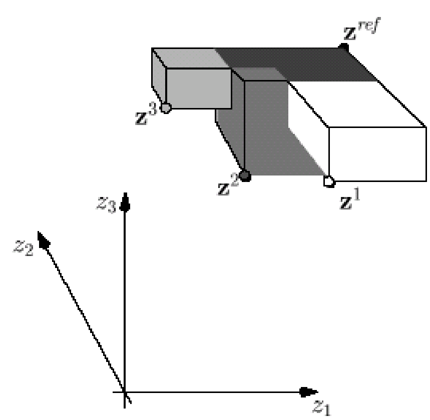


Fig. (2). Size of dominated space, S index, for the set $A = \{z_1, z_2, z_3\}$.

There are many disadvantages to this metric. It strictly requires defining some upper boundary of the region within which all feasible points will lie(z^{ref}). The choice of this boundary does affect the ordering of nondominated sets[10]. Also, S metric has a very large computational time. Veldhuizen [10] suggests that metric S can be misleading if the Pareto optimal front is non-convex.

(2.2) The Error ratio (ER) metric

Error ratio metric has been introduced by Veldhuizen [10] and it is defined as $ER = \frac{\sum_{i=1}^{Q} e_i}{Q}$ where Q is the number of vectors in the approximation set Z; $e_i = 0$ if vector *i* is in Z^* and 1 otherwise. Lower values of the error ratio represent better nondominated sets. ER is the proportion of non true Pareto points in Z. It is a reference metric using Z^* as reference set. It induces a total ordering of nondominated sets.

It is worth mentioning here that although a member of Q is a Pareto optimal, if that solution does not exist in Z^*, it may be counted in this metric as a non Pareto optimal solution. Another drawback is that if no member of Q is in the Pareto optimal set, it does not distinguish the relative closeness of any set Q from Z^*. In Fig. (3) the nondominated set A on the left has an error of zero. The set A on the right has an error ratio of 4/7 but the set on the right is clearly better,

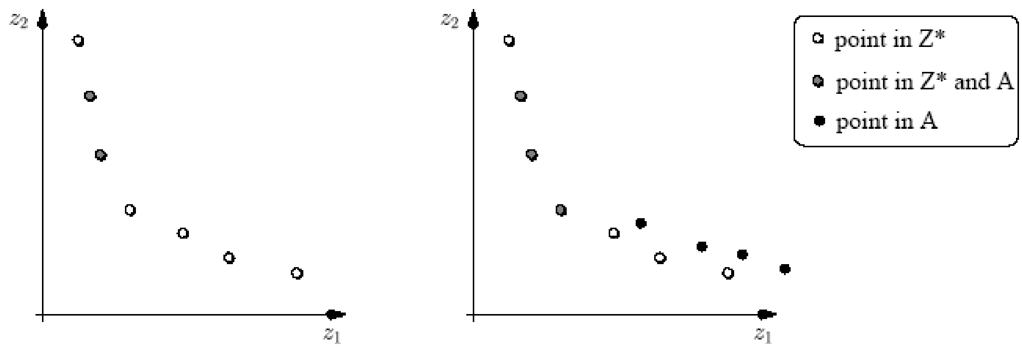


Fig. (3). An example when error ratio metric.

(2.3) The C metric

The metric C, like metric S, was introduced by Zitzler in [12] . Using metric C, two sets of nondominated solutions can be compared to each other.

Let $A, B \subseteq X$ be two sets of decision vectors. The function C maps the ordered pair (A, B) to the interval [0,1]:

$$C(A,B) = \frac{|\{b \in B \mid a \in A : a \succeq b|}{|B|}$$

The value C(A, B) =1 means that all decision vectors in B are weakly dominated by A. The opposite, C(A,B)=0, represents the situation when none of the points in B are weakly dominated by A.

There are situations when the metric C cannot decide if an obtained front is better than the other. Let us suppose that front1 correspond to a set A and front 2 to a set B. In Fig. (4), the surface covered by the front 1 is equal to the surface covered by the front 2 but front 2 is closer to the Pareto optimal front than front 1. In this situation (and in other situations similar with this) the C metric is not applicable. To eliminate this shortcoming a new metric "D metric" was proposed.

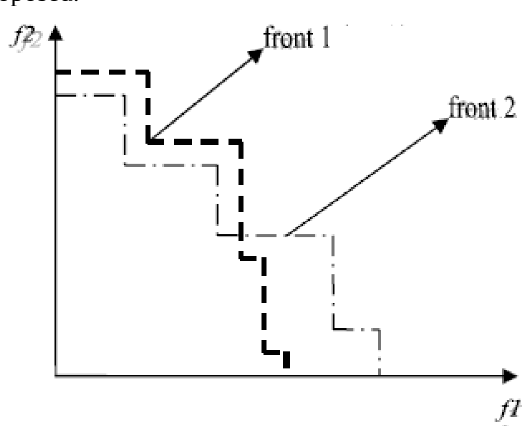


Fig. (4). Metric C can not decide between front 1 and front 2.

(2.4) The D metric

Let $A, B \subseteq X$ be two sets of decision vectors. the size of the space dominated by A and not dominated by B (regarding the objective space) is denoted D(A, B) and is defined as: $D(A, B) = S(A + B) - S(B)$,

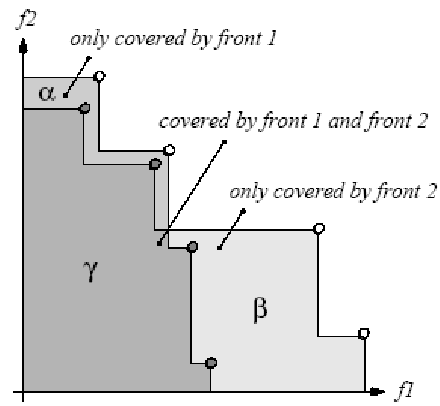


Fig. (5). Example of difference between C metric and D metric.

Metric D can be used to solve the inconvenience of Fig. (4). Consider the notations from Fig. (5). By applying metric D as follows

$$S(A,B) = \alpha + \beta + \gamma, \quad S(A) = \alpha + \gamma, \quad S(B) = \gamma + \beta$$

The metric D for this example is expressed below.

$$D(A,B) = \alpha, \quad D(B,A) = \beta$$

Then $D(A,B) < D(B,A)$ it results that front 2 dominates front 1. Zitzler [13] suggest that (ideally) the D metric is used in combination with the S metric where the values may be normalized by a reference volume V, where (for a maximization problem) V is given by:

$$V = \prod_{i=1}^{k} (f_i^{\max} - f_i^{\min})$$

$f_i^{\max}$ *and* $f_i^{\min}$ represent the maximum respectively minimum value for the objective f_i Thus, the value $D'(A,B) = \frac{D(A,B)}{V}$ represents the relative size of the region (objective space) dominated by A and not dominated by B.

3. New Metric For Convergence

In this section we propose a new metric for evaluate the convergence to the Pareto set with no requirement to know the true Pareto set or other reference points.

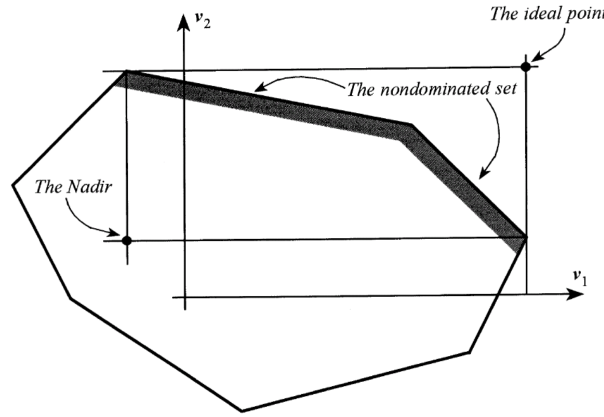


Fig. (6). Ideal and Nadir concept.

Definition 1. (Ideal objective vector $z^* \in R^k$)

An objective vector minimizing each of the objective functions is called an ideal (perfect) objective vector. The component z_i^* of the ideal objective vector $z^* \in R^k$ are defined by minimizing each of the objective functions individually subject to the constraints, that is, by solving

$$\min f_i(x)$$
$$s.t.\ x \in S, \quad For\ i = 1,..,k$$

From the ideal objective vector we obtain the lower bounds of the Pareto optimal set for each objective function as depicted in Fig. (6).

Definition 2. (Nadir objective vector $z_* \in R^k$)

Nadir objective vector is the upper bounds of the optimal set, that is, the components of the nadir objective vector (imperfect or anti-ideal).

The procedure for computing convergence metric is given in the next steps:

Step 0: Calculate the ideal and Nadir objective vector Z^*, Z_*

Step 1. Identify the nondominated set Z(t) of population P(t).

Step 2. Rank nondominated set Z(t) according to $f_1(\cdot)$

Step 3. t=0;

Step 4. Repeat :

Step 5. t=t+1

Step 6. AI=Area from observed Pareto set to ideal point =sum(area of triangles with vertices (Z_t, Z_{t+1}, Z^*))

Step 7. AN= Area from observed Pareto set to nadir point =sum(area of triangle with vertices (Z_t, Z_{t+1}, Z_*))

Step 8. Termination: Until t=size of Z(t)

Step 9: Calculate the metric as follows

$$\textbf{Proposed metric}: A = \frac{AI}{AI + AN}$$

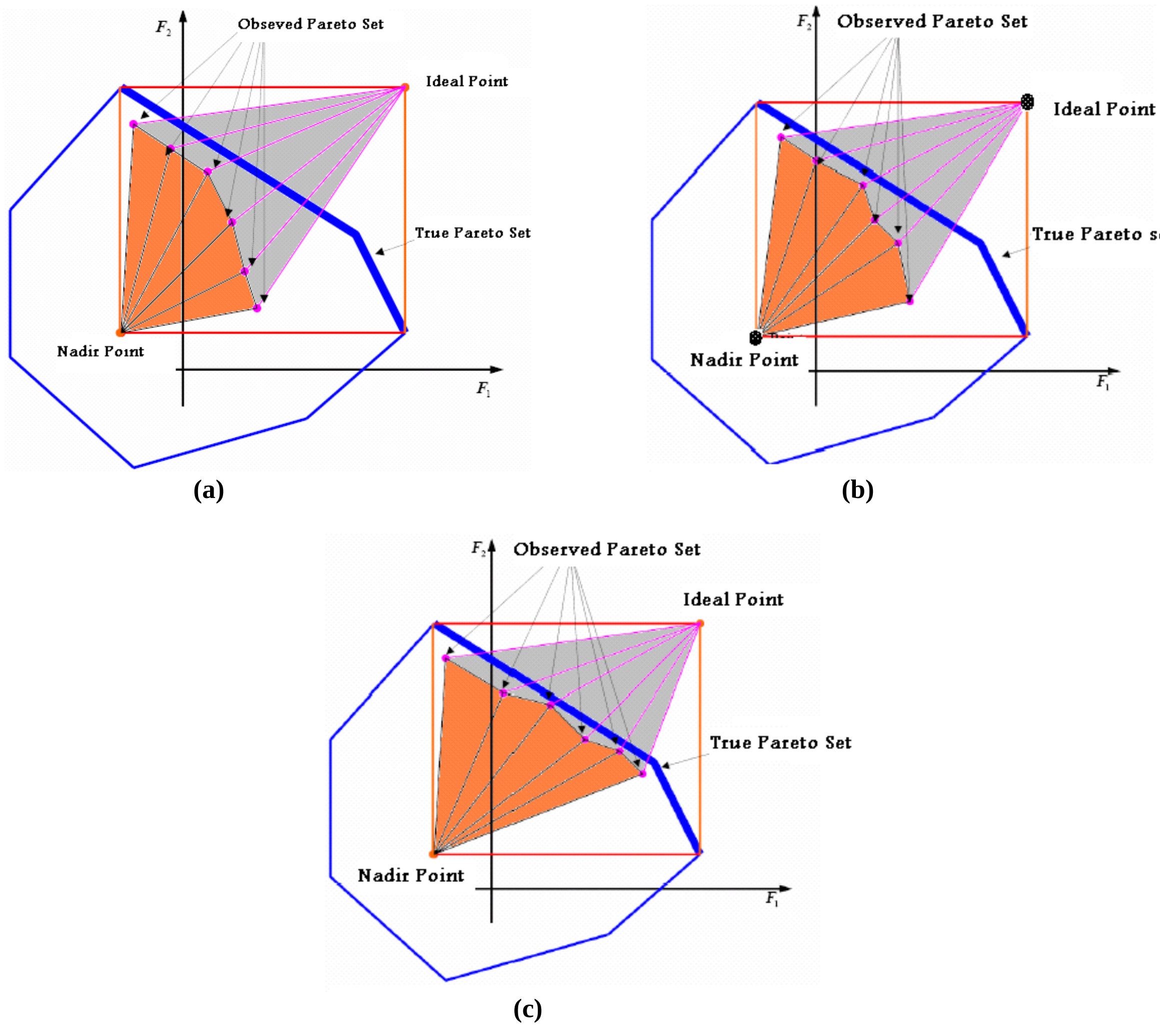


Fig. (7). Three different observed Pareto set.

The proposed metric use the measuring formula with reference information (ideal and nadir points) as obtained during the calculation, to enable the designer monitor and potentially improve the performance of a multiobjective optimization method.

The relative small values of metric A indicate that approximation set Z_2 are closed to the Pareto optimal set than the other ser Z_1 in Fig. (7), also the approximation set Z_3 is closed to Pareto than Z_2 and Z_1.

The proposed metric is easy to understand and easy to calculate, there is no requirement of knowing true Pareto optimal front. This metric enable a designer to monitor the quality of an observed Pareto solution set as obtained by a multiobjective optimization algorithm and guide it towards the Pareto optimal set or compare the quality of observed Pareto solution sets as reported by different multiobjective optimization methods.

Also, evaluating approximations for Pareto set is useful when performing experimental comparisons of different multiple objective heuristic algorithms, or when defining stopping rules of multiple objective heuristic algorithms

4. Numerical Analysis

This section is devoted to the discussion of effects of different problems[6] and different Pareto set on the proposed metric.

An iterative Co-evolutionary algorithm for multiobjective optimization problem (IT-CEMOP)[8] was applied to different problem. In order to validate the proposed metric graphical representation, statistical analysis of the experimental results are presented. Table (1) lists the parameter setting used in the algorithm for all runs.

Table (1). GA parameters.

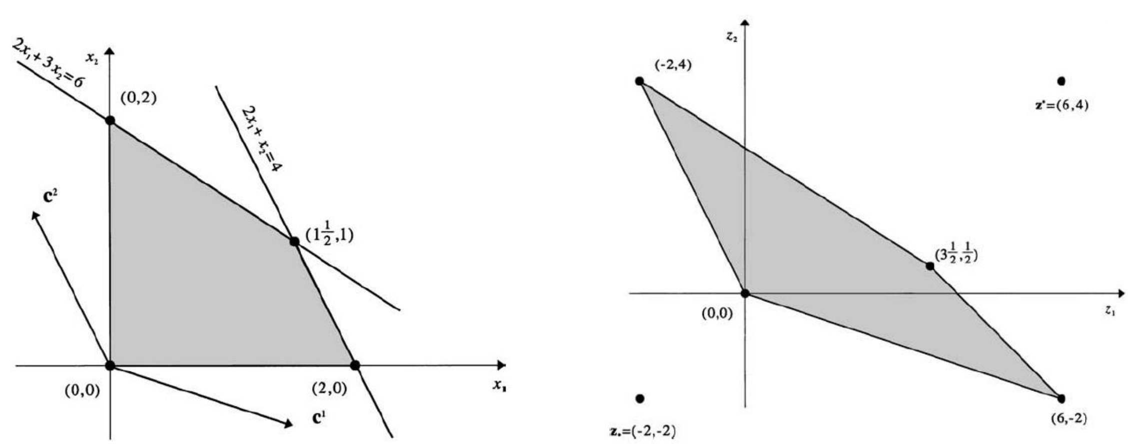
Population size (N)	10
No. of Generation	100
Crossover probability	0.92
Mutation probability	0.02
Selection operator	Roulette Wheel
Crossover operator	BLX-α
Mutation operator	Polynomial mutation
Relative tolerance ε	10e-2

Problem 1: $Max\ \ z_1 = 3x_1 - x_2, \quad Max\ \ z_2 = -x_1 + 2x_2$

$$S.t.$$

$$2x_1 + 3x_2 \le 6, \quad 2x_1 + x_2 \le 4, \quad x_1, x_2 \ge 0.$$

The feasible regions of this solution in both decision and criterion spaces are shown in Fig. (8). The efficient set is $E = \gamma((0,2),(3/2,1)) \cup \gamma((3/2,1),(2,0))$ and the Pareto set is $P = \gamma((-2,4),(7/2,1/2)) \cup \gamma((7/2,1/2),(6,-2))$. The ideal and Nadir point are $Z^* = (6,4)$ with $x^* = (3\frac{1}{5}, 3\frac{3}{5})$ and $Z_* = (-2,-2)$ with $x_* = (-1\frac{1}{5}, -1\frac{3}{5})$.



(a) (b)

Fig. (8). Graph of Problem 1: feasible region in (a) decision space and (b) criterion space.

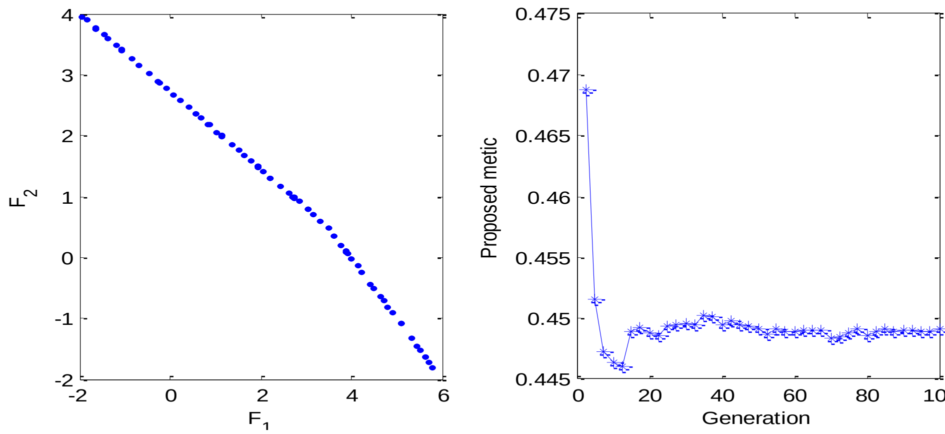


Fig. (9). Experimental results of Problem 1: (a) Pareto set after 100 generation (b) Proposed metric vs. generation.

For different observed Pareto set at different generation, graphical representation of the proposed metric is presented in Fig. (9). It is obvious that after a few generations the metric deviation is small $\|A - A_{mean}\|_\infty$ which coincides with the fact that genetic algorithm convergence to the promising region of solution in first few generations. Also, the metric values are constant after generation 85.

Problem 2: $Max\ z_1 = 4x_1 + x_2,\ \ Max\ z_2 = x_1 + 5x_2$

$$S.t.$$

$$2x_1 + 3x_2 \le 12,\quad 2x_1 + x_2 \le 8,\qquad x_1, x_2 \ge 0.$$

The feasible regions of this solution in both decision and criterion spaces are shown in Fig. (10). The efficient set is $E = \gamma((0,4),(3,2)) \cup \gamma((3,2),(4,0))$ and the Pareto set is $P = \gamma((4,20),(14,13)) \cup \gamma((14,13),(16,4))$. The ideal and Nadir point are $Z^* = (16,20)$ with $x^* = (\frac{60}{19}, \frac{64}{19})$ and $Z_* = (4,4)$ with $x_* = (\frac{16}{19}, \frac{12}{19})$.

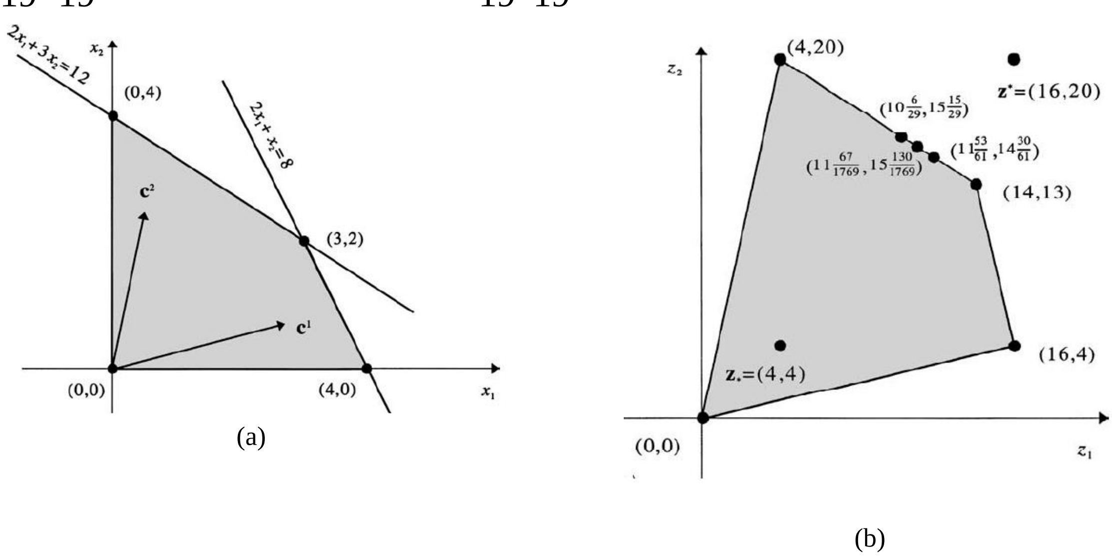


Fig. (10). Graph of problem 2: feasible region in (a) decision space and (b) criterion space.

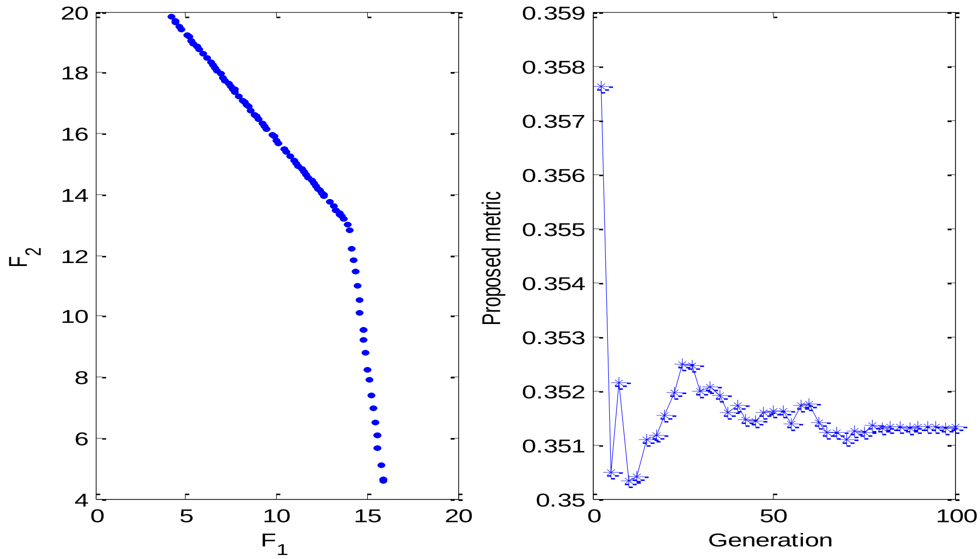


Fig. (11). Experimental results of Problem 2: (a) Pareto set after 100 generation (b) Proposed metric vs. generation.

For different observed Pareto set at different generation, graphical representation of the proposed metric are presented in Fig. (11). It is obvious that after a few generations the metric deviation is small $\|A - A_{mean}\|_\infty$ .Also, the metric values are constant after generation 80.

Problem 3: $Max\ \mathbf{z_1 = x_1} + x_2,\ \ Max\ \mathbf{z_2 = x_1} + (5/2)x_2$

$S.t.$

$$x_1 + 3x_2 \le 18,\ \ x_1 + 2x_2 \le 13,\ \ \ 4x_1 + 3x_2 \le 32,\ \ \ x_1, x_2 \ge 0.$$

The feasible regions of this solution in both decision and criterion spaces are shown in Fig. (12). The efficient set is $E = \gamma((9,15),(8,15\frac{1}{2}))$ and the Pareto set is $P = \gamma((-2,4),(7/2,1/2)) \cup \gamma((7/2,1/2),(6,-2))$. The ideal and Nadir point are $Z^* = (9,15\frac{1}{2})$ with $x^* = (4\frac{2}{3},4\frac{1}{3})$ and $Z_* = (8,15)$ with $x_* = (3\frac{1}{3},4\frac{2}{3})$.

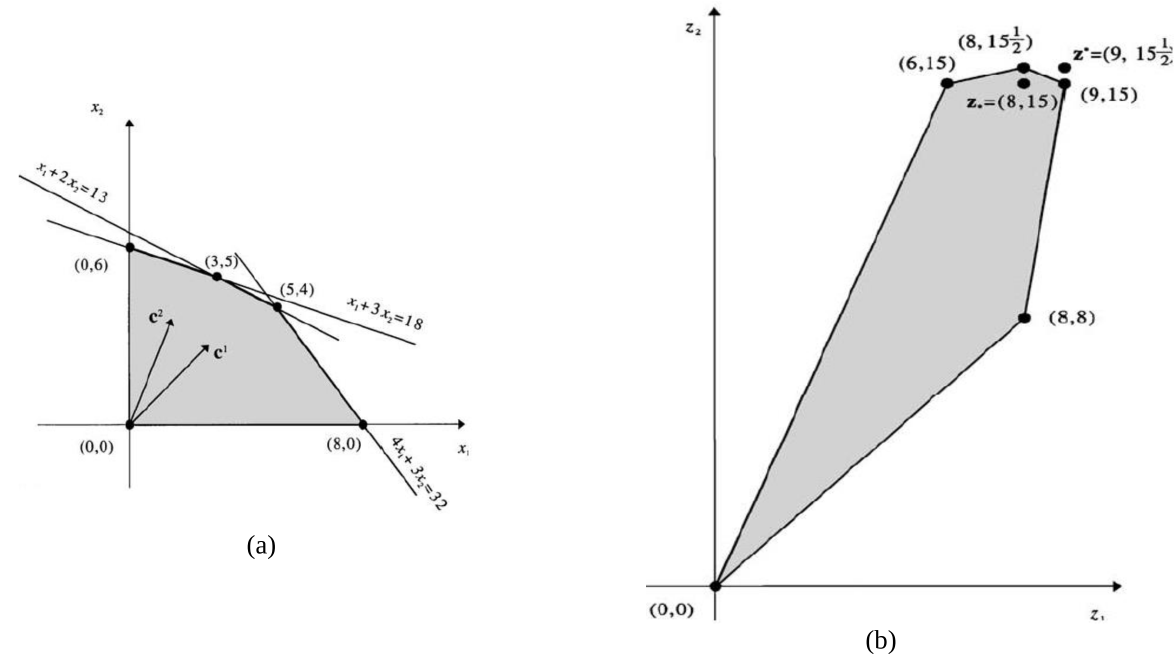


Fig (12). Graph of problem 3: feasible region in (a) decision space and (b) criterion space.

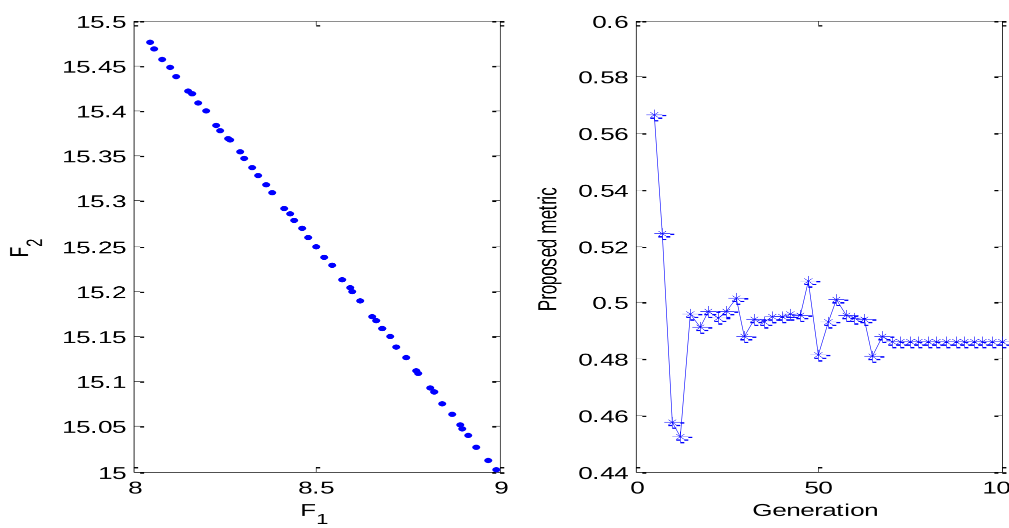


Fig. (13). Experimental results of Problem 3: (a) Pareto set (b) Proposed metric vs. generation.

For different observed Pareto set at different generation, graphical representation of the proposed metric are presented in Fig. (13).The metric values are constant after generation 65.

Problem 4: $\max\ \mathbf{z_1} = \mathbf{2x_1} + x_2, \quad \max\ \mathbf{z_2} = \mathbf{x_1} + 5x_2$

$$S.t.$$

$$x_1 + 3x_2 \le 18, \quad x_1 + 2x_2 \le 13, \quad 4x_1 + 3x_2 \le 32, \quad x_1, x_2 \ge 0.$$

The feasible regions of this solution in both decision and criterion spaces are shown in Fig. (14). The efficient set is $E = \gamma((8,0),(5,4)) \cup \gamma((5,4),(3,5)) \cup \gamma((3,5),(0,6))$ and the Pareto set is $P = \gamma((16,8),(14,25)) \cup \gamma((14,25),(11,28)) \cup \gamma((11,28),(6,30))$. The ideal and Nadir point are $Z^* = (16,30)$ with $x^* = (5\frac{5}{9}, 4\frac{8}{9})$ and $Z_* = (6,8)$ with $x_* = (2\frac{4}{9}, 1\frac{1}{9})$.

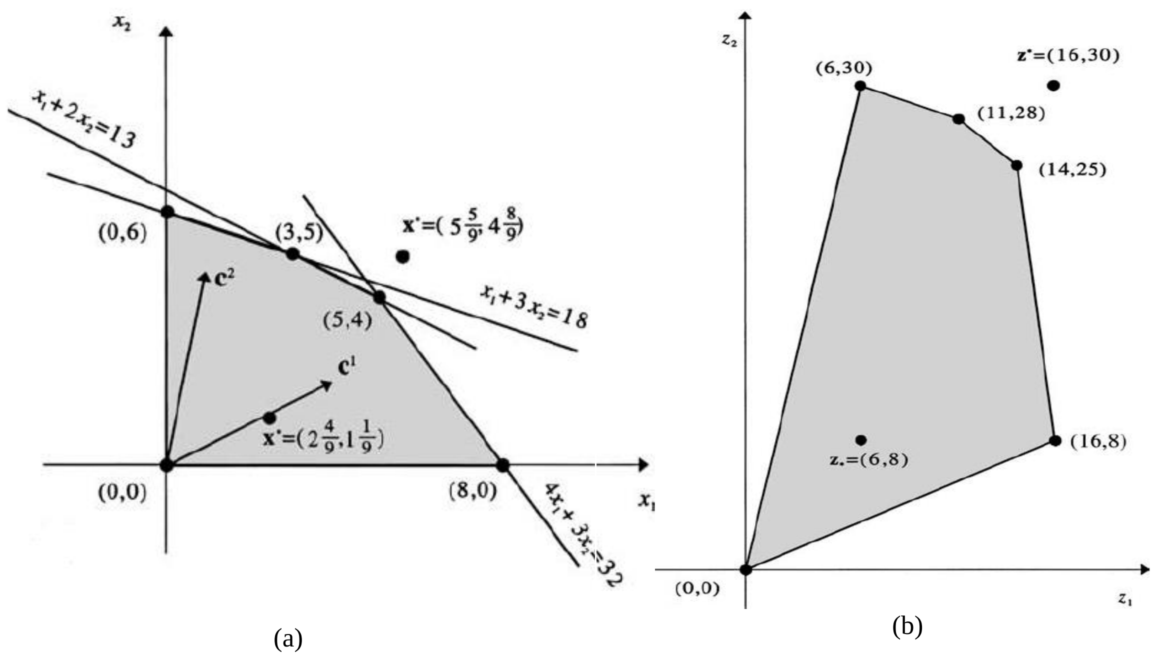


Fig. (14). Graph of problem 4: feasible region in (a) decision space and (b) criterion space.

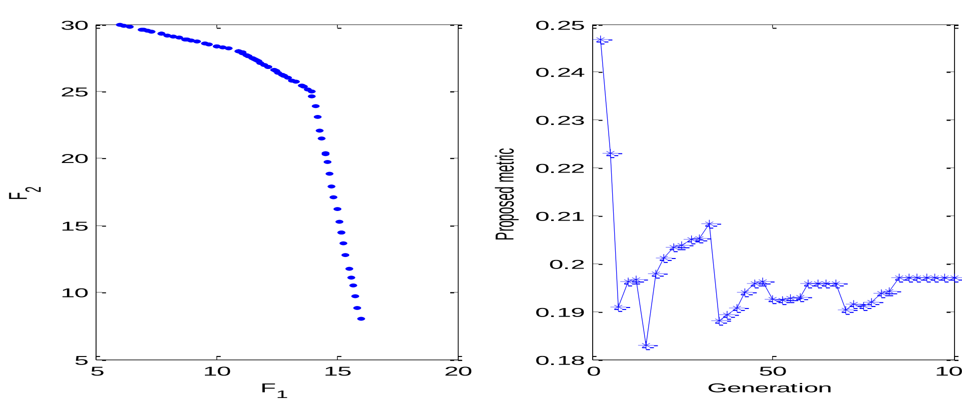


Fig. (15). Experimental results of Problem 4: (a) Pareto set after 100 generation (b) Proposed metric vs. generation.

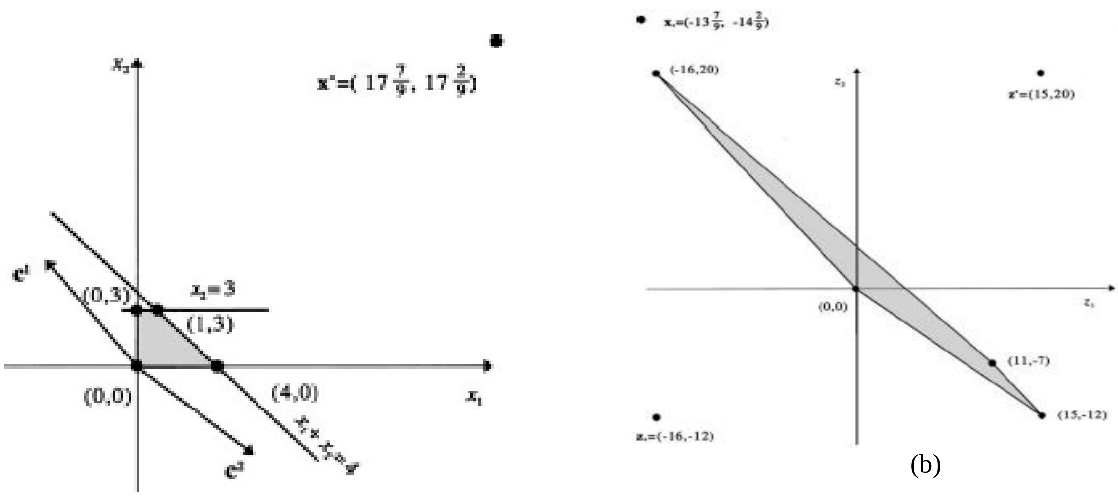
For different Pareto set at different generation, graphical representation of the proposed metric are presented in Fig. (15) It is declare after 86 generation the metric values are constant.

Problem 5: Max z_1=-4x_1+5x_2 , Max z_2=5x_1-4x_2

S.t.

$$x_2 \le 3 \;, \quad x_1+x_2 \le 4, \quad x_1, x_2 \ge 0.$$

The feasible regions of this solution in both decision and criterion spaces are shown in Fig. (16). The efficient set is $E = \gamma((0,3),(1,3)) \cup \gamma((1,3),(4,0))$ and the Pareto set is $P = \gamma((15,-12),(11,-7)) \cup \gamma((11,-7),(-16,20))$. The ideal and Nadir point are $Z^* = (15,20)$ with $x^* = (17\frac{7}{9}, 17\frac{2}{9})$ and $Z_* = (-16,-12)$ with $x_* = (-13\frac{7}{9}, 14\frac{2}{9})$.



(a) (b)

Fig. (16). Graph of problem 5: feasible region in (a) decision space and (b) criterion space.

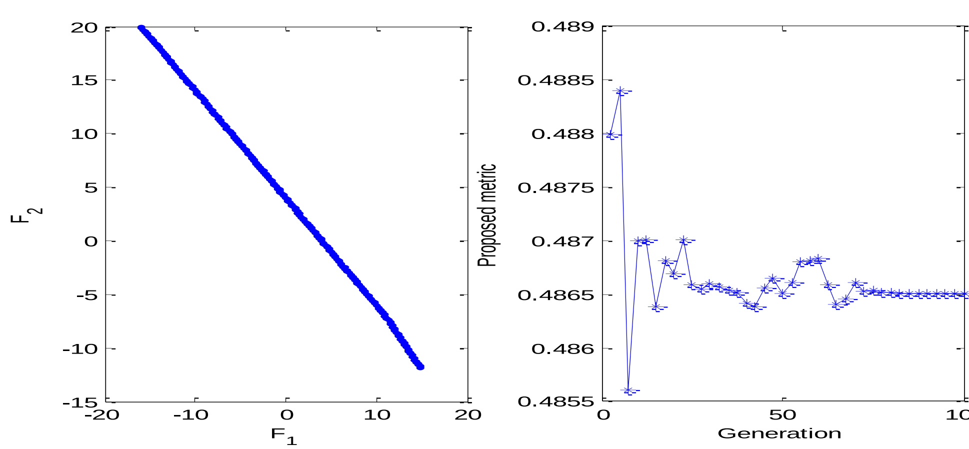


Fig. (17). Experimental results of Problem 5: (a) Pareto set after 100 generation (b) Proposed metric vs. generation.

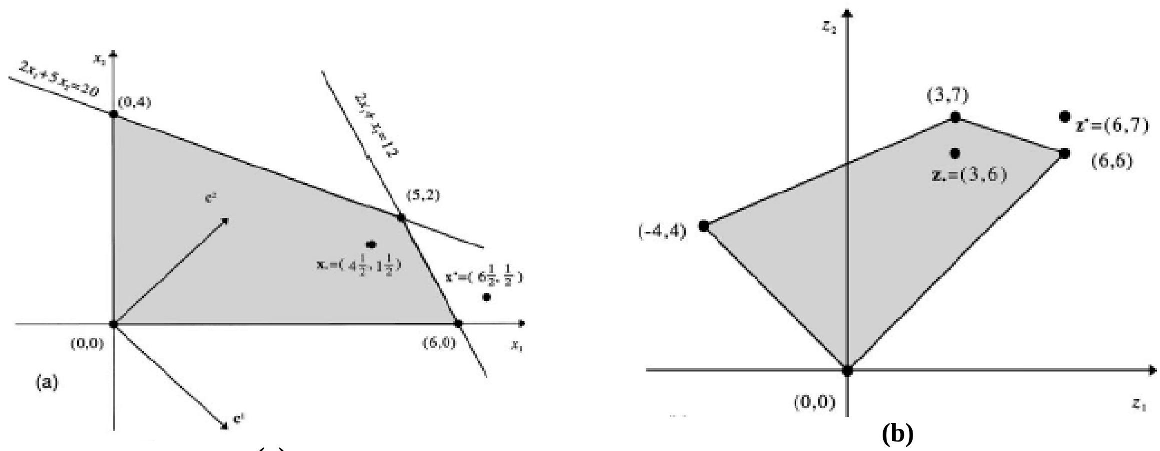
For different Pareto set at different generation, graphical representation of the proposed metric are presented in Fig. (17). It is obvious after 64 generation the metric values are constant.

Problem 6: $Max\ z_1 = x_1 - x_2\ ,Max\ z_2 = x_1 + x_2$

$$S\,t.$$

$$x_1 + x_2 \le 12\ ,\quad 2x_1 + 5x_2 \le 20,\quad x_1, x_2 \ge 0.$$

The feasible regions of this solution in both decision and criterion spaces are shown in Fig. (18). The efficient set is $E = \gamma((5,2),(6,0))$ and the Pareto set is $P = \gamma((3,7),(6,6))$. The ideal and Nadir point are $Z^* = (6,7)$ with $x^* = (6\frac{1}{2},\frac{1}{2})$ and $Z_* = (3,6)$ with $x_* = (4\frac{1}{2},1\frac{1}{2})$.



(a)

Fig. (18). Graph of problem 6: feasible region in (a) decision space and (b) criterion space.

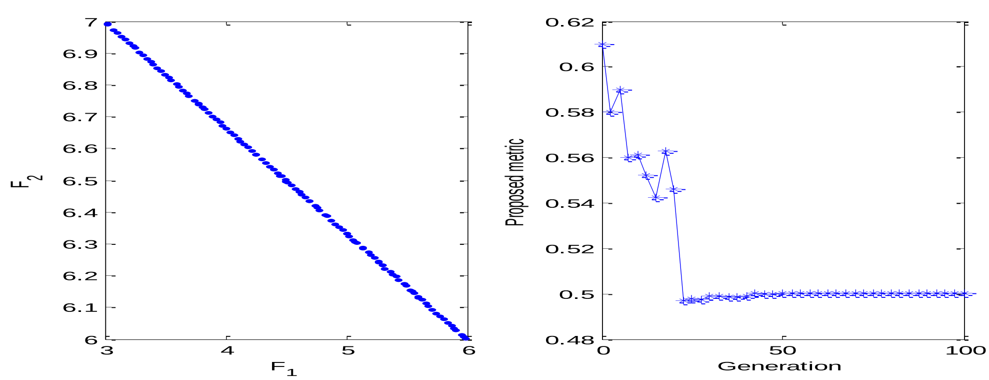


Fig. (19). Experimental results of Problem 6: (a) Pareto set after 100 generation (b) Proposed metric vs. generation.

For different Pareto set at different generation, graphical representation of the proposed metric are presented in Fig. (19) It is obvious that after 47 generation the metric values are constant.

As the result the proposed metric can be used as a stopping criteria which save computational time for the previous problems as in table (2).

Table (2). Percentage of computational time saving.

Problem	Percentage saving	Problem	Percentage saving
Problem 1	$\frac{100-85}{100}=15\%$	Problem 4	$\frac{100-86}{100}=14\%$
Problem 2	$\frac{100-80}{100}=20\%$	Problem 5	$\frac{100-64}{100}=36\%$
Problem 3	$\frac{100-65}{100}=35\%$	Problem 6	$\frac{100-47}{100}=53\%$

5. Conclusion

Many metrics have been proposed in the last years. Most of them calculate the convergence to an obtained set of solutions to the true Pareto front. We can not say that one metric is the best. Some of them are preferred to the others by considering the computation complexity. For different classes of problems different types of metrics can be preferred

The performance metric presented in this paper provide a means to measure the goodness of an observed Pareto solution set. The following are the significant characteristics of this metric :

(a) There is no requirement of knowing the true Pareto optimal front.

(b) It is easy to understand and easy to implement.

(c) It can be used to compare the goodness of observed Pareto solutions as reported by multiobjective optimization method.

(d) It can be used as stopping criteria in evolutionary algorithms.

(e) By using this metric, the quality of various multiobjective optimization methods can be compared against one another.

Experimental results show that the proposed metric may hopefully be a good monitoring through the algorithm proceed toward Pareto optimal solution.

6. Reference

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قياس مدى كفاءة الحلول الناتجة من طرق التقييم المختلفة لمشاكل الأمثلية المتعددة الأهداف

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ملخص البحث. من السهل إيجاد مقياس Metric يسمح بمقارنة الحلول المختلفة لمشاكل الأمثلية ذات الهدف الواحد وتصنيفها حتى مع عدم معرفة الحل الأمثل. في حين أن من الصعب إيجاد مثل هذا لمقياس في حالة مشاكل الأمثلية ذات الأهداف المتعددة Multiobjective نظراً لان الحل هنا هو مجموعة كبيرة من الحلول (Pareto optimal front) . ويوجد العديد من الطرق المستخدمة في حل مشاكل الأمثلية ذات الأهداف المتعددة ولمقارنة تلك الطرق المختلفة أو لقياس مدى أداء أي منها يوجد العديد من المقاييس Metrics ولابد لتلك المقاييس من أن تُمكن المصمم من متابعة وملاحظة مدى كفاءة الحلول الناتجة ومقارنة الحلول الناتجة من طرق مختلفة لتقييم آي تلك الطرق أفضل .

في هذا البحث تم تقديم وتوظيف مقياس Metric جديد اعتمد فى بنائه على حساب كلا من Ideal point , Nadir point بحيث يمكن استخدام هذا المقياس وسيله يتم بها مقارنه الحلول الناتجه عن طرق مختلفه لتحديد ايها أفضل وكذلك تم استخدامه كوسيلة سريعة وجيدة لتقييم مدى التقدم في اتجاه الحل الامثل بحيث يمكن استخدامه كوسيله توقف Stopping criteria وتم توظيف هذا المقياس من خلال مجموعة مختلفه من التطبيقات وأكدت النتائج أن هذا المقياس يعتبر وسيله جيده وفاعله لتقييم الحل ومدى التقدم فى اتجاه الوصول الى الحل الامثل True Pareto solution.