Journal of Natural Sciences and Mathematics
Qassim University, Vol. 8, No. 2, pp 153-166 (July 2015/Ramadan 1436H.)

On Modified Laguerre Matrix Polynomials

Ayman Shehata

Department of Mathematics, College of Science and Arts, Unaizah, Qassim University, Qassim, Kingdom of Saudi Arabia.

Department of Mathematics, Faculty of Science, Assiut University, Assiut 71516, Egypt.

E-mail: drshehata2006@yahoo.com

Abstract. This work is devoted to the study of modified Laguerre matrix polynomials. Hypergeometric matrix function representations, integral representation and an explicit expression for the modified Laguerre matrix polynomials are investigated. Finally, identities for power series for the modified Laguerre matrix polynomials are derived.

Keywords and phrases*:* Modified Laguerre matrix polynomials; Hypergeometric matrix function; Generating matrix functions.

AMS Mathematics Subject Classification(2010): 15A60, 33C45, 33E20.

1. Introduction and Preliminaries

Orthogonal matrix polynomials is an emergent field whose development is reaching critical or important results from both the theoretical and practical points of view. As in the corresponding problem for scalar polynomials, the problem of the development of matrix polynomials in series of Laguerre matrix polynomials requires some new results about the matrix operational calculus not available in the literature. Indeed, in recent papers, matrix polynomials have significant emergent. An extension to the matrix framework of the classical families of Laguerre, Hermite, Gegenbauer, Rice's, Rainville's, Humbert and Konhauser polynomials have been introduced in [1, 3, 4, 8, 10, 11, 12, 13, 14, 15, 16, 17] for matrices in $C^{N\times N}$ whose eigenvalues are all situated in the right open half-plane. The reasons of interest for this family of modified Laguerre polynomials are due to their intrinsic mathematical importance.

Our main aim in this paper is to define and study a class of modified Laguerre matrix polynomials from a different point of view, starting from a generalization of the generating matrix function. The structure of the paper is organized as follows: In Section 2 a definition of modified Laguerre matrix polynomials are given starting from the generating matrix function, and the hypergeometric matrix representations, matrix differential recurrence relations and an expansion of polynomials for modified Laguerre matrix polynomials are obtained. Finally, the study of developments of the finite summations and new properties for the modified Laguerre matrix polynomials is obtained and discusses their interesting properties in Section 3.

Throughout this paper, for a matrix A in $C^{N\times N}$, its spectrum $\sigma(A)$ denotes the set of all eigenvalues of A. We say that a matrix A in $C^{N\times N}$ is a positive stable matrix [5, 10, 16] if $Re(z) > 0$ for all $z \in \sigma(A)$. Its two-norm will be denoted by $\| A\|_2$ and is defined by

$$\| A\|_2 = \sup_{x\neq 0} \frac{\| Ax\|_2}{\| x\|_2}$$

where for a vector x in C^N, $\| x\|_2 = (x^T x)^{\frac{1}{2}}$ is the Euclidean norm of x. Furthermore the identity matrix and the null matrix or zero matrix in $C^{N\times N}$ will be denoted by I and O, respectively.

Fact 1.1 If $f(z)$ and $g(z)$ are holomorphic functions of the complex variable z, which are defined in an open set Ω of the complex plane, and A, B are matrices in $C^{N\times N}$ with $\sigma(A)\subset\Omega$ and $\sigma(B)\subset\Omega$, such that $AB=BA$, then from the properties of the matrix functional calculus in [2], it follows that

$$f(A)g(B)=g(B)f(A).$$

Definition 1.1 Let P be a positive stable matrix in $C^{N\times N}$. Then the Gamma matrix function $\Gamma(P)$ has been defined in [5], as follows

$$\Gamma(P)=\int_0^{\infty} e^{-t}t^{P-I}dt; \quad t^{P-I}=\exp((P-I)\ln t). \tag{1.1}$$

Fact 1.2 [5] We recall that the reciprocal scalar Gamma function $\Gamma^{-1}(z)=\dfrac{1}{\Gamma(z)}$ is an entire function of the complex variable z and thus for any matrix A in $C^{N\times N}$, the image of $\Gamma^{-1}(z)$ acting on A is denoted by $\Gamma^{-1}(A)$ and is well-defined matrix. Furthermore, if A is a matrix in $C^{N\times N}$ such that

$$A+nI \quad \text{is an invertible matrix for all integers } n\ge 0. \tag{1.2}$$

Then the Pochhammer symbol or shifted factorial is defined by

$$(A)_n=A(A+I)...(A+(n-1)I)=\Gamma(A+nI)\Gamma^{-1}(A); \quad n\ge 1; \quad (A)_0=I. \tag{1.3}$$

Fact 1.3 For any matrix A in $C^{N\times N}$, the authors exploit the following relation (see [6]):

$$(1-x)^{-A}={}_1F_0(A;-;x)=\sum_{n=0}^{\infty}\frac{1}{n!}(A)_n x^n; \quad |x|<1 \tag{1.4}$$

where $(A)_n$ is defined by (1.3).

Definition 1.2 [7, 9] If A is a matrix in $C^{N\times N}$ such that the condition is satisfied:

$$-k\notin\sigma(A), \quad \text{for every integer } k>0 \tag{1.5}$$

and λ is a complex parameter with $Re(\lambda) > 0$, then the Laguerre matrix polynomials are defined as

$$L_n^{(A,\lambda)}(x) = \sum_{k=0}^{n} \frac{(-1)^k (A+I)_n [(A+I)_k]^{-1} \lambda^k x^k}{k!(n-k)!}, n \geq 0 \tag{1.6}$$

and

$$(1-t)^{-(A+I)} \exp(\frac{-\lambda xt}{1-t}) = \sum_{n=0}^{\infty} L_n^{(A,\lambda)}(x) t^n, x, t \in R, |t| < 1. \tag{1.7}$$

Lemma 1.1 [7] For matrices $A(k,n)$ and $B(k,n)$ in $C^{N\times N}$ where $n \geq 0$, $k \geq 0$ the following relations are satisfied:

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} B(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} B(k,n-k). \tag{1.8}$$

Similarly, we can write

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n} B(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} B(k,n+k). \tag{1.9}$$

2. On Modified Laguerre Matrix Polynomials

Let A be a matrix in $C^{N\times N}$ satisfying the condition (1.5) and λ is a complex parameter with $Re(\lambda) > 0$, then the n^{th} modified Laguerre matrix polynomials $f_n^{(A,\lambda)}(x)$ is defined by the generating matrix function

$$W(x,t,A) = (1-t)^{-A} e^{\lambda xt} = \sum_{n=0}^{\infty} f_n^{(A,\lambda)}(x) t^n; x, t \in R, |t| < 1. \tag{2.1}$$

From (1.4), (1.8) and (2.1), we get

$$(1-t)^{-A}e^{\lambda xt} = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(A)_k(\lambda x)^n}{n!k!}t^{n+k} = \sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(A)_k(\lambda x)^{n-k}}{k!(n-k)!}t^n. \tag{2.2}$$

In order to change x^{n-k} to x^k, we replace k with $n-k$. Thus we have

$$f_n^{(A,\lambda)}(x) = \sum_{k=0}^{n}\frac{(A)_k(\lambda x)^{n-k}}{k!(n-k)!} = \sum_{k=0}^{n}\frac{(A)_{n-k}(\lambda x)^k}{k!(n-k)!}. \tag{2.3}$$

It is clear that $f_n^{(A,\lambda)}(0) = \dfrac{(A)_n}{n!}$.

From the relation (1.2), one obtains

$$\frac{1}{(n-k)!}I = \frac{(-1)^k(-n)_k}{n!}I = \frac{(-1)^k(-nI)_k}{n!}; \quad 0 \le k \le n. \tag{2.4}$$

Using (2.4), the explicit representation (2.3) becomes

$$f_n^{(A,\lambda)}(x) = \sum_{k=0}^{n}\frac{(-1)^k(-nI)_k(A)_k(\lambda x)^{n-k}}{k!n!} = \frac{(\lambda x)^n}{n!}\,{}_2F_0\left(-nI, A; -; -\frac{1}{\lambda x}\right).$$

From (1.3), it is easy to see that

$$(A)_{n-k} = (-1)^k(A)_n[(I-A-nI)_k]^{-1}; \quad 0 \le k \le n. \tag{2.5}$$

Using (2.5) in (2.3), we have

$$\begin{aligned} f_n^{(A,\lambda)}(x) &= \sum_{k=0}^{n}\frac{(-1)^k(-1)^k(-nI)_k(A)_n[(I-A-nI)_k]^{-1}(\lambda x)^k}{k!n!} \\ &= \frac{(A)_n}{n!}\,{}_1F_1(-nI; I-nI-A; \lambda x). \end{aligned}$$

From the above expressions, we have the proof of the following theorem.

Theorem 2.1 Let A be a matrix in $C^{N\times N}$ satisfying the condition (1.5) and λ is a complex parameter with $Re(\lambda)>0$, the hypergeometric matrix representations for the modified Laguerre matrix polynomials are given by:

$$f_n^{(A,\lambda)}(x) = \frac{(\lambda x)^n}{n!}\, {}_2F_0\left(-nI, A; -; -\frac{1}{\lambda x}\right) \tag{2.6}$$

and

$$f_n^{(A,\lambda)}(x) = \frac{(A)_n}{n!}\, {}_1F_1\left(-nI; I-nI-A; \lambda x\right). \tag{2.7}$$

Note that: This hypergeometric matrix function ${}_1F_1$ is an entire function, by this mean the modified Laguerre matrix polynomials is an entire function.

Next, we derive the integral representation for the modified Laguerre matrix polynomials:

Theorem 2.2 Let A be a matrix in $C^{N\times N}$ satisfying the condition (1.5) and λ is a complex parameter with $Re(\lambda)>0$, then we have

$$f_n^{(A,\lambda)}(x) = \frac{1}{n!}\Gamma^{-1}(A)\int_0^\infty t^{A-I}(\lambda x - t)^n e^{-t}dt. \tag{2.8}$$

Proof. From (1.1) and (1.4) the right-hand side of equation (2.8) can be written in the form

$$\begin{aligned}
&\frac{1}{n!}\sum_{k=0}^{n}\frac{(-1)^k n!}{k!(n-k)!}(\lambda x)^{n-k}\,\Gamma^{-1}(A)\int_0^\infty t^{A+(k-1)I}e^{-t}dt \\
&= \frac{1}{n!}\sum_{k=0}^{n}\frac{(-1)^k n!}{k!(n-k)!}(\lambda x)^{n-k}\,\Gamma^{-1}(A)\Gamma(A+kI) \\
&= \frac{1}{n!}\sum_{k=0}^{n}\frac{(-1)^k (\lambda x)^{n-k}}{k!}(-nI)_k\,(A)_k
\end{aligned}$$

which in view of (2.6) gives us the left-hand side of equation (2.8) and this complete the proof.

Theorem 2.3 Let A be a matrix in $C^{N\times N}$ satisfying the condition (1.5). The modified Laguerre matrix polynomials verify the following properties

$$D_x^r f_n^{(A,\lambda)}(x) = \lambda^r f_{n-r}^{(A,\lambda)}(x);\ D_x^r = \frac{d^r}{dx^r},\ 0 \le r \le n. \tag{2.9}$$

Proof. Differentiating (2.1) with respect to x, we get

$$\sum_{n=0}^{\infty} D_x^r f_n^{(A,\lambda)}(x) t^n = (\lambda t)^r (1-t)^{-A} e^{\lambda x t} = \lambda^r \sum_{n=0}^{\infty} f_{n-r}^{(A,\lambda)}(x) t^n .$$

Thus the proof of Theorem **2.3** is completed.

Corollary 2.1 The modified Laguerre matrix polynomials satisfy the following matrix differential recurrence relation

$$x \frac{d}{dx} f_n^{(A,\lambda)}(x) = (A + nI + \lambda\, xI) f_n^{(A,\lambda)}(x) - (n+1) f_{n+1}^{(A,\lambda)}(x). \tag{2.10}$$

Proof. From (2.1) and (2.9), it is easy to see (2.10).

Next, we use the expansion of the modified Laguerre matrix polynomials together with their interesting properties to prove the following result.

Theorem 2.4 Let A be a matrix in $C^{N\times N}$ satisfying the condition (1.5), then

$$(\lambda x)^n I = n! \sum_{k=0}^{n} \frac{(-A)_k}{k!} f_{n-k}^{(A,\lambda)}(x). \tag{2.11}$$

Proof. Since

$$e^{\lambda x t} I = (1-t)^A \sum_{n=0}^{\infty} f_n^{(A,\lambda)}(x) t^n = \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{(-A)_k}{k!} f_{n-k}^{(A,\lambda)}(x) t^n ,$$

we have (2.11).

3 Some Properties for Modified Laguerre Matrix Polynomials

The generating matrix functions of section 2 lead to certain simple finite sum properties for the modified Laguerre matrix polynomials and give some interesting properties satisfied by these polynomials. We first state our result as in the following:

Theorem 3.1 If A and B are matrices in $C^{N\times N}$ satisfying the condition (1.5), such that A and B are commuting matrices, then the finite summation for modified Laguerre matrix polynomials are

$$f_n^{(A,\lambda)}(x)=\sum_{k=0}^{n}\frac{(A-B)_k}{k!}\,y^k\,f_{n-k}^{(B,\lambda)}(x) \tag{3.1}$$

and

$$f_n^{(A+B,\lambda)}(x+y)=\sum_{k=0}^{n}f_k^{(A,\lambda)}(x)\,f_{n-k}^{(B,\lambda)}(y). \tag{3.2}$$

Proof. Equation (2.1) implies that

$$\begin{aligned}(1-t)^{-A}e^{\lambda xt} &= (1-t)^{-(A-B)}(1-t)^{-B}e^{\lambda xt}=(1-t)^{-(A-B)}\sum_{n=0}^{\infty}f_n^{(B,\lambda)}(x)t^n\\ &=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(A-B)_k}{k!}\,f_n^{(B,\lambda)}(x)t^{n+k}=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(A-B)_k}{k!}\,f_{n-k}^{(B,\lambda)}(x)t^n,\end{aligned}$$

which leads to (3.1).

On the other hand from (2.1), we have

$$(1-t)^{-A}e^{\lambda xt}(1-t)^{-B}e^{\lambda yt}=\sum_{n=0}^{\infty}\sum_{k=0}^{n}f_k^{(A,\lambda)}(x)f_{n-k}^{(B,\lambda)}(y)t^n.$$

Hence, we get (3.2).

Corollary 3.1 If A is a matrix in $C^{N\times N}$ satisfying the condition (1.5), then we have the following property

$$f_n^{(2A,\lambda)}(2x) = \sum_{k=0}^{n} f_{n-k}^{(A,\lambda)}(x)\, f_k^{(A,\lambda)}(x). \tag{3.3}$$

Proof. The equation (3.3) follows directly from (3.2) by putting $A = B$ and $x = y$.

Next, we derive additional formula, which can be obtained from the generating matrix function (2.1).

Theorem 3.2 Let A be a matrix in $C^{N\times N}$ satisfying the condition (1.5) and let λ be a complex number with $Re(\lambda) > 0$, then the modified Laguerre matrix polynomials satisfy the addition formula as follows:

$$f_n^{(A,\lambda)}(x+y) = \sum_{k=0}^{n} \frac{y^k}{k!}\, f_{n-k}^{(A,\lambda)}(x). \tag{3.4}$$

Proof. Starting from (2.1) and using (1.8), we get

$$\sum_{n=0}^{\infty} f_n^{(A,\lambda)}(x+y)t^n = (1-t)^{-A} e^{\lambda(x+y)t} = e^{\lambda\, yt} \sum_{n=0}^{\infty} f_n^{(A,\lambda)}(x)t^n$$
$$= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{y^k}{k!}\, f_n^{(A,\lambda)}(x)t^{n+k} = \sum_{n=0}^{\infty}\sum_{k=0}^{n} \frac{y^k}{k!}\, f_{n-k}^{(A,\lambda)}(x)t^n.$$

which leads to (3.4).

Theorem 3.3 Let A be a matrix in $C^{N\times N}$ satisfying the condition (1.5) and let λ be a complex number with $Re(\lambda) > 0$, then we have

$$\sum_{k=0}^{n} f_{n-k}^{(A,\lambda)}(x)\, f_k^{(-A,\lambda)}(-x) = \begin{cases} I, & n = 0; \\ O, & n = 1,2,3,.... \end{cases} \tag{3.5}$$

Proof. From (2.1) and (1.10), we obtain

$$\sum_{n=0}^{\infty} f_n^{(A,\lambda)}(x)t^n \sum_{k=0}^{\infty} f_k^{(-A,\lambda)}(-x)t^k = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} f_n^{(A,\lambda)}(x)\, f_k^{(-A,\lambda)}(-x)t^{n+k}$$
$$= \sum_{n=0}^{\infty}\sum_{k=0}^{n} f_{n-k}^{(A,\lambda)}(x)\, f_k^{(-A,\lambda)}(-x)t^n = (1-t)^{-A} e^{xt} (1-t)^{A} e^{-xt} = I,$$

which gives (3.5).

Theorem 5.4 Let n and k be positive integers with $k \geq 2$, then we have the identity

$$\sum_{s=0}^{n} \frac{((k-1)A)_s}{s!} f_{n-s}^{(A,\lambda)}(kx) = \sum_{n=n_1+n_2+\ldots+n_k} f_{n_1}^{(A,\lambda)}(x) f_{n_2}^{(A,\lambda)}(x) \ldots f_{n_k}^{(A,\lambda)}(x). \tag{3.6}$$

Proof. From (2.1), (1.4) and (1.8), we have

$$\begin{aligned} W(kx,t,kA) &= (1-t)^{-kA} e^{k\lambda xt} = (1-t)^{-(k-1)A}(1-t)^{-A} e^{k\lambda xt} \\ &= (1-t)^{-(k-1)A} \sum_{n=0}^{\infty} f_n^{(A,\lambda)}(kx) t^n = \sum_{n=0}^{\infty}\sum_{s=0}^{\infty} \frac{((k-1)A)_s}{s!} f_n^{(A,\lambda)}(kx) t^{n+s} \\ &= \sum_{n=0}^{\infty} \left(\sum_{s=0}^{n} \frac{((k-1)A)_s}{s!} f_{n-s}^{(A,\lambda)}(kx) \right) t^n \end{aligned} \tag{3.7}$$

On the other hand, using (2.1), we have

$$\begin{aligned} (1-t)^{-kA} e^{k\lambda xt} &= \left[(1-t)^{-A} e^{\lambda xt}\right]^k = \left[\sum_{n=0}^{\infty} f_n^{(A,\lambda)}(x) t^n\right]^k \\ &= \sum_{n=0}^{\infty} \left[\sum_{n_1+n_2+\ldots+n_k=n} f_{n_1}^{(A,\lambda)}(x) f_{n_2}^{(A,\lambda)}(x) \ldots f_{n_k}^{(A,\lambda)}(x) \right] t^n. \end{aligned} \tag{3.8}$$

Combining (3.7) and (3.8) and comparing the coefficients of t^n, we have the desired relation.

In a similar way as in the proof of Theorem 2.1, we derive in the following results.

Theorem 3.5 For A_1, A_2,..., A_k are matrices in $C^{N\times N}$ satisfying the condition $(-z) \notin \sigma(A_i), \forall z \in Z^+$, and let λ_i be a complex parameter with $Re(\lambda_i) > 0$ for $i = 1,2,\ldots,k$. For any positive integers n and k with $k \geq 2$, we have

$$\sum_{s=0}^{n} \frac{(\mathbf{A}-A_i)_s}{s!} f_{n-s}^{(A_i,\lambda_1+\lambda_2+\ldots+\lambda_k)}(kx)$$
$$= \sum_{n=n_1+n_2+\ldots+n_k} f_{n_1}^{(A_1,\lambda_1)}(kx) f_{n_2}^{(A_2,\lambda_2)}(kx)\ldots f_{n_k}^{(A_k,\lambda_k)}(kx); i=1,2,\ldots,k, \tag{3.9}$$

where $\mathbf{A}=A_1+A_2+\ldots+A_k$, and the matrices A_1, A_2,..., A_k are assumed to be commutative.

Theorem 3.6 Let A_1, A_2,..., A_k be commutative matrices in $C^{N\times N}$ satisfying the condition $(-z)\notin\sigma(A_i), \forall z\in Z^+$, and let λ_i be a complex parameter with $Re(\lambda_i)>0$ for $i=1,2,\ldots,k$. Let n and k be positive integers with $k\geq 2$, then we have

$$\sum_{s=0}^{n} \frac{(\mathbf{A}-A_i)_s}{s!} f_{n-s}^{(A_i,\lambda_i)}\left(\frac{\lambda_1 x_1+\lambda_2 x_2+\ldots+\lambda_k x_k}{\lambda_i}\right)$$
$$= \sum_{n=n_1+n_2+\ldots+n_k} f_{n_1}^{(A_1,\lambda_1)}(x_1) f_{n_2}^{(A_2,\lambda_2)}(x_2)\ldots f_{n_k}^{(A_k,\lambda_k)}(x_k); i=1,2,\ldots,k, \tag{3.10}$$

where $\mathbf{A}=A_1+A_2+\ldots+A_k$.

In a forthcoming work, we will consider the problems of a unified approach to the theory of new orthogonal matrix polynomials by following the technique discussed in this paper.

Definition 3.1 *Let* A *be a matrix in* $C^{N\times N}$ *satisfying the spectral conditions* $Re(\lambda)>0$ *for each eigenvalue* $\lambda\in\sigma(A)$. *For* $n\geq 0$, *the Charlier matrix polynomials* $C_n^{(A,\lambda)}(x)$ *of degree* n *is defined as*

$$C_n^{(A,\lambda)}(x) = \sum_{k=0}^{n} (-1)^k \binom{n}{k}\binom{x}{k} k!(\lambda A)^{-k} \tag{3.11}$$

and generating matrix function of the Charlier matrix polynomials by

$$\sum_{n=0}^{\infty} \frac{C_n^{(A,\lambda)}(x)t^n}{n!} = \exp(-\lambda\, At)(1+t)^x. \tag{3.12}$$

Definition 3.2 Let A, B, C be matrices in $C^{N\times N}$, A satisfying the condition (1.5) and λ is a complex parameter with $Re(\lambda) > 0$, and $B + nI$ is an invertible matrix for every integer $n \geq 0$ and $Re(b) \neq 0$ for all $b \in \sigma(B)$, we will define the generalized modified Laguerre matrix polynomials $f_n^{(A,B,C,\lambda)}(x)$ in the form

$$f_n^{(A,B,C,\lambda)}(x) = \frac{B^n (A)_n}{n!} \, {}_1F_1(-nI; A; \lambda x C B^{-1}). \tag{3.13}$$

Acknowledgements

(1) The Author expresses their sincere appreciation to Prof. Dr. Nassar H. Abdel-All (Department of Mathematics, Faculty of Science, Assiut University, Assiut 71516, Egypt), Dr. M. S. Metwally, (Department of Mathematics, Faculty of Science (Suez), Suez Canal University, Egypt) and Dr. Mahmoud Tawfik Mohamed, (Department of Mathematics and Science, Faculty of Education(New Valley), Assiut University, New Valley, EL-Kharga 72111, Egypt) for their kinds interests, encouragements, helps, suggestions, comments them and the investigations for this series of papers.

(2) The author would like to thank the referees for their valuable comments and suggestions which have led to the better presentation of the paper.

References

[1] Çekim, B., Altin, A., and Aktaş, R. Some relations satisfied by orthogonal matrix Ppolynomials. *Hacettepe J. Math. Statistics*, **40 (2)** (2011), 241-253.

[2] Dunford, N., and Schwartz, J.T. *Linear Operators, part I, General Theory*. Interscience Publishers, INC. New York, 1957.

[3] Khan, S., and Hassan, N.A.M. 2-variable Laguerre matrix polynomials and Lie-algebraic techniques. *J. Phys. A.*, **43(23)** (2010), 235204 (21pp).

[4] Jódar, L., Company, R., and Navarro, E. Laguerre matrix polynomials and systems of second order differential equations. *Appl. Numer. Math.*, **15** (1994), 53-63.

[5] Jódar, L., and Cortés, J.C. Some properties of gamma and beta matrix functions. *Appl. Math. Lett.*, **11** (1998), 89-93.

[6] Jódar, L., and Cortés, J.C. On the hypergeometric matrix function. *J. Computational Appl. Math.*, **99** (1998), 205-217.

[7] Jódar, L., and Sastre, J. On Laguerre matrix polynomials. *Utilitas Math.*, **53** (1998), 37-48.

[8] Rainville, E.D. *Special Functions*. Macmillan, New York, 1962.

[9] Sastre J., and Jódar, L. On Laguerre matrix polynomials series. *Utilitas Math.*, **71** (2006), 109-130.

[10] Shehata, A. On Tricomi and Hermite-Tricomi matrix functions of complex variable. *Communications Math. Applications*, **2 (2-3)** (2011), 97 - 109.

[11] Shehata, A. On Rice's matrix polynomials. *Afrika Matematika*, **25(3)** (2014), 757-777.

[12] Shehata, A. On Rainville's matrix polynomials. *Sylwan Journal*, **158(9)**, (2014), 158-178.

[13] Shehata, A. New kinds of Hypergeometric matrix functions. *British Journal of Mathematics and Computer Science*, **5(1)** (2015), 92-102.

[14] Shehata, A. On a new family of the extended generalized Bessel-type matrix polynomials, *Mitteilungen Klosterneuburg Journal*, **65(2)** (2015), 100-121.

[15] Shehata, A. Some relations on Gegenbauer matrix polynomials. *Review of Computer Engineering Research*, **2(1)** (2015)1-21.

[16] Upadhyaya, L.M., and Shehata, A. A new extension of generalized Hermite matrix polynomials, *Bulletin Malaysian Mathematical Sci. Soc.*, **38(1)** (2015), 165–179.

[17] Varma, S., Çekim, B., and Taşdelen Yesildal, F. On Konhauser matrix polynomials. *Ars Comb.*, **100** (2011), 193-204.

"حول كثيرات حدود لاجير المعدلة المصفوفية"

أيمن شحاتة محمد أحمد

قسم الرياضيات – كلية العلوم والآداب بعنيزة (اقسام الطلاب) – جامعـة القصيم – المملكة العربية السعودية.

قسم الرياضيات – كلية العلوم – جامعة أسيوط – مصر.

ملخص البحث.يخصص هذا العمل لدراسة كثيرات حدود لاجير المعدلة المصفوفة. وحصلنا علي تمثيل للدالة فوق الهندسية المصفوفية و الصيغة التكاملية لها وعبرنا عن مفكوك لكثيرات حدود لاجير المعدلة المصفوفية . وأخيرا ونشتق الخواص لمتسلسلات القوي لكثيرات حدود لاجير المعدلة المصفوفية.