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Positive solutions of a singular 4th order two-point boundary value problem

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Abstract. This paper investigates the existence and nonexistence of positive solutions for nonlinear higher order boundary value problem. This can be achieved with the help of suitable theorems by means of the fixed point theory for studying a nonlinear real functions and continuous differential equation in the Banach space of a bounded and closed interval. We show that the mentioned 4^{th} order differential equation has at least one positive solution.

Keywords: *Positive solutions; singular 4th order; two-point boundary value problem; fixed-point theorem.*

1.Introduction

To study the existence and nonexistence of a positive solution for the following higher order boundary value problem with the help of a suitable theorems, we show the solvability nonlinear 4^{th} order differential equation. On the other hand, the existent results of positive solutions for integer order differential equations have been studied by several researchers (see[8-11] and others references), but, as far as we know, only a few papers consider the BVP for higher order nonlinear differential equations in the Banach space of real functions and continuous on a bounded and closed interval, (see[1,3,5], and others references). So, the aim of this paper is to fill this gap. In this paper we obtain the existence and nonexistence of a positive solution for the BVP (3.1) and (3.2) in the Banach space. The results presented in this paper seem to be new and original. They generalize several results obtained up till now in the study of nonlinear differential equations of several types.

2.Notation, definition and auxiliary results

Theorem 2.1 [Agarwal et.al [2], and Li, S [4]]

Assume that U is a relatively open subset of convex set K in Banach space E. Let $N:\overline{U}\rightarrow K$ be a compact map with $o\in U$ .Then either

(i) N has a fixed point in $\overline{U}$;or

(ii)There is a $u\in U$ and a $\lambda\in(0,1)$ such that $u=\lambda\, N u$.

Definition 2.1 An operator is called completely continuous if it is continuous and maps bounded sets into pre-compacts.

Definition 2.2 *Let E be a real Banach space. A nonempty closed convex set* $K\subset E$ *is called cone of E if it satisfies the following conditions:*

(i) $x\in K, \sigma\geq o$ *implies* $\sigma x\in K$ *;*

(ii) $x\in K, -x\in K$ *implies* $x=o$.

3. Main results

In this section,we will study the existence and nonexistence of positive solutions for thenonlinear boundary value problem:

$$u^{(4)}(t)-\alpha(t)\,f(t,u(t))=0, \qquad a\prec t\prec b \tag{3.1}$$

$$u(a)=u(b)=u''(b)=u'''(b)=0, \tag{3.2}$$

Theorem 3.1.Under conditions (3.2), equation (3.1) has a unique solution.

Proof: Applying the Laplace transform to equation (3.1) we get

$$s^4\bar{u}(s)-s^3u(0)-s^2u'(0)-s\ u''(0)-u'''(0)=\bar{y}(s) \tag{3.3}$$

Where $\bar{u}(s)$ and $\bar{\mathrm{y}}(\mathrm{s})$ is the Laplace transform of $u(t)$ and $\mathrm{y(t)}$ respectively. Laplace inversion of Eq.(3.4) gives the final solution as:

$$s^4\bar{u}(s)=s^3u(0)+s^2u'(0)+s\ u''(0)+u'''(0)+\bar{y}(s)$$

$$\bar{u}(s)=s^3\frac{u(0)}{s^4}+s^2\frac{u'(0)}{s^4}+s\ \frac{u''(0)}{s^4}+\frac{u'''(0)}{s^4}+\frac{\bar{y}(s)}{s^4}$$

$$\bar{u}(s)=s^3\frac{A}{s^4}+s^2\frac{B}{s^4}+s\ \frac{C}{s^4}+\frac{E}{s^4}+\frac{\bar{y}(s)}{s^4}$$

$$u(t)=A+Bt+C\frac{t^2}{2!}+E\frac{t^3}{3!}+\int_0^t\frac{(t-s)^3}{3!}y(s)ds$$

$$0=A+Ba+C\frac{a^2}{2!}+E\frac{a^3}{3!}+\int_0^a\frac{(a-s)^3}{3!}y(s)ds$$

$$0=A+Bb+C\frac{b^2}{2!}+E\frac{b^3}{3!}+\int_0^b\frac{(b-s)^3}{3!}y(s)ds$$

$$B=-C\frac{a^2-b^2}{2!(a-b)}-E\frac{a^3-b^3}{3!(a-b)}-\frac{1}{(a-b)}\int_0^a\frac{(a-s)^3}{3!}y(s)ds+\frac{1}{(a-b)}\int_0^b\frac{(b-s)^3}{3!}y(s)ds$$

$$B=-\frac{a^2-b^2}{2!(a-b)}\int_0^b sy(s)ds+\frac{a^3-b^3}{3!(a-b)}\int_0^b y(s)ds-\frac{1}{(a-b)}\int_0^a\frac{(a-s)^3}{3!}y(s)ds+$$
$$\frac{1}{(a-b)}\int_0^b\frac{(b-s)^3}{3!}y(s)ds$$

$$A=\frac{a^2-b^2}{2!(a-b)}a\int_0^b sy(s)ds-\frac{a^3-b^3}{3!(a-b)}a\int_0^b y(s)ds+\frac{1}{(a-b)}a\int_0^a\frac{(a-s)^3}{3!}y(s)ds-$$
$$\frac{1}{(a-b)}a\int_0^b\frac{(b-s)^3}{3!}y(s)ds-\frac{a^2}{2!}\int_0^b sy(s)ds+\frac{a^3}{3!}\int_0^b y(s)ds-\int_0^a\frac{(a-s)^3}{3!}y(s)ds$$

$$u(t)=\frac{a^2-b^2}{2!(a-b)}a\int_0^b sy(s)ds-\frac{a^3-b^3}{3!(a-b)}a\int_0^b y(s)ds+\frac{1}{(a-b)}a\int_0^a\frac{(a-s)^3}{3!}y(s)ds-$$
$$\frac{1}{(a-b)}a\int_0^b\frac{(b-s)^3}{3!}y(s)ds-\frac{a^2}{2!}\int_0^b sy(s)ds+\frac{a^3}{3!}\int_0^b y(s)ds-\int_0^a\frac{(a-s)^3}{3!}y(s)ds-$$
$$\frac{a^2-b^2}{2!(a-b)}\int_0^b tsy(s)ds+\frac{a^3-b^3}{3!(a-b)}\int_0^b ty(s)ds-\frac{1}{(a-b)}\int_0^a t\frac{(a-s)^3}{3!}y(s)ds+\frac{1}{(a-b)}\int_0^b t\frac{(b-s)^3}{3!}y(s)ds-$$
$$\int_0^b\frac{t^2}{2!}sy(s)ds-\int_0^b\frac{t^3}{3!}y(s)ds+\int_0^t\frac{(t-s)^3}{3!}y(s)ds.$$

$$u(t)=\frac{a^2-b^2}{2!(a-b)}(a-t)\int_0^b sf(t,u(s))ds-\frac{a^3-b^3}{3!(a-b)}(a-t)\int_0^b f(t,u(s))ds+\frac{1}{(a-b)}(a-t)\int_0^a\frac{(a-s)^3}{3!}f(t,u(s))ds-$$
$$\frac{1}{(a-b)}(a-t)\int_0^b\frac{(b-s)^3}{3!}f(t,u(s))ds-\frac{(a^2-t^2)}{2!}\int_0^b sf(t,u(s))ds+\frac{(a^3-t^3)}{3!}\int_0^b f(t,u(s))ds$$
$$-\int_0^a\frac{(a-s)^3}{3!}f(t,u(s))ds+\int_0^t\frac{(t-s)^3}{3!}f(t,u(s))ds$$

The proof is complete.

Defining $T:X\rightarrow X$ as:

$$Tu(t)=\frac{a^2-b^2}{2!(a-b)}(a-t)\int_0^b sf(t,u(s))ds-\frac{a^3-b^3}{3!(a-b)}(a-t)\int_0^b f(t,u(s))ds+$$
$$\frac{1}{(a-b)}(a-t)\int_0^a\frac{(a-s)^3}{3!}f(t,u(s))ds-\frac{1}{(a-b)}(a-t)\int_0^b\frac{(b-s)^3}{3!}f(t,u(s))ds-\frac{(a^2-t^2)}{2!}\int_0^b sf(t,u(s))ds$$
$$+\frac{(a^3-t^3)}{3!}\int_0^b f(t,u(s))ds-\int_0^a\frac{(a-s)^3}{3!}f(t,u(s))ds+\int_0^t\frac{(t-s)^3}{3!}f(t,u(s))ds$$

Where X=C[0,1] is the Banach space endowed with the supper norm. We have the following result for operator T.

Lemma 3.1 Assume that $f:[a,b]\times R\rightarrow R$ is continuous function, then T is completely continuous operator.

Proof: It is easy to see that T is continuous. For $u\in M=\{u\in X;\|u\|\leq l,\ l\succ 0\}$,we

$$|Tu(t)| = \left| \begin{array}{l} \frac{a^2-b^2}{2!(a-b)}(a-t)\int_0^b sf(t,u(s))ds - \frac{a^3-b^3}{3!(a-b)}(a-t)\int_0^b f(t,u(s))ds + \\ \frac{1}{(a-b)}(a-t)\int_0^a \frac{(a-s)^3}{3!} f(t,u(s))ds - \\ \frac{1}{(a-b)}(a-t)\int_0^b \frac{(b-s)^3}{3!} f(t,u(s))ds - \frac{(a^2-t^2)}{2!}\int_0^b sf(t,u(s))ds \\ + \frac{(a^3-t^3)}{3!}\int_0^b f(t,u(s))ds - \int_0^a \frac{(a-s)^3}{3!} f(t,u(s))ds + \int_0^t \frac{(t-s)^3}{3!} f(t,u(s))ds \end{array} \right|$$

$$\begin{aligned} |Tu(t)| \le {} & \frac{a^2-b^2}{2!(a-b)}(a-t)\int_0^b s|f(t,u(s))|ds + \frac{a^3-b^3}{3!(a-b)}(a-t)\int_0^b |f(t,u(s))|ds + \\ & \frac{1}{(a-b)}(a-t)\int_0^a \frac{(a-s)^3}{3!}|f(t,u(s))|ds + \frac{1}{(a-b)}(a-t)\int_0^b \frac{(b-s)^3}{3!}|f(t,u(s))|ds + \\ & \frac{(a^2-t^2)}{2!}\int_0^b s|f(t,u(s))|ds + \frac{(a^3-t^3)}{3!}\int_0^b f|(t,u(s))|ds + \\ & + \int_0^a \frac{(a-s)^3}{3!}|f(t,u(s))|ds + \int_0^t \frac{(t-s)^3}{3!}|f(t,u(s))|ds \end{aligned}$$

$$\begin{aligned} |Tu(t)| \le {} & \frac{a^2-b^2}{2!(a-b)}L(a-t)\int_0^b sds + \frac{a^3-b^3}{3!(a-b)}(a-t)L\int_0^a ds + \\ & \frac{1}{(a-b)}L(a-t)\int_0^a \frac{(a-s)^3}{3!}ds + \frac{1}{(a-b)}L(a-t)\int_0^b \frac{(b-s)^3}{3!}ds + \frac{(a^2-t^2)}{2!}L\int_0^b sds \qquad (3.4) \\ & + \frac{(a^3-t^3)}{3!}L\int_0^b ds + L\int_0^a \frac{(a-s)^3}{3!}ds + L\int_0^t \frac{(t-s)^3}{3!}ds. \end{aligned}$$

$$|Tu(t)| \le \frac{a^2-b^2}{2!(a-b)}L(a-t)\left[\frac{s^2}{2}\right]_0^b + \frac{a^3-b^3}{3!(a-b)}(a-t)L[s]_0^a +$$
$$\frac{1}{(a-b)}L(a-t)\left[\frac{(a-s)^4}{4!}\right]_0^a + \frac{1}{(a-b)}L(a-t)\left[\frac{(b-s)^4}{4!}\right]_0^b + \frac{(a^2-t^2)}{2!}L\left[\frac{s^2}{2}\right]_0^b$$
$$+\frac{(a^3-t^3)}{3!}L[s]_0^b + L\left[\frac{(a-s)^4}{4!}\right]_0^a + L\left[\frac{(t-s)^4}{4!}\right]_0^t$$
$$\le \frac{(a^2-b^2)L}{(2!)(a-b)}(a-t)\left[\frac{b^2}{2}\right] + \frac{(a^3-b^3)L}{(3!)(a-b)}(a-t)[a] - \frac{L}{(a-b)}(a-t)\left[\frac{a^4}{4!}\right] - \tag{3.5}$$
$$\frac{L}{(a-b)}(a-t)\left[\frac{b^4}{4!}\right] + \frac{(a^2-t^2)L}{(2!)}\left[\frac{b^2}{2}\right] + \frac{(a^3-t^3)L}{(3!)}[b] - L\frac{a^4}{4!} - L\frac{t^4}{4!}$$

where $L = \max_{0\le t\le 1, \|u\|\le 1} |f(t,u(t))| + 1,$

So T(M) is bounded. Next we shall show the equicontinuity of $\overline{T(M)}$. $\forall \varepsilon \succ 0, t_1 \prec t_2 \in [0,1]$.

Let

$$,\eta \prec \left\{\frac{2!(2)\varepsilon}{6Lb^2}\right\}, \quad \gamma \prec \left\{\frac{\varepsilon}{Lb}\right\}, \quad \rho \prec \left\{\frac{2!(2)(a-b)\varepsilon}{6(a^2-b^2)b^2L}, \frac{3!(a-b)\varepsilon}{6(a^3-b^3)Lb}, \frac{4!(a-b)\varepsilon}{6L(a^4+b^4)}\right\}, \tag{3.6}$$
$$\mu < \left\{\frac{4!\varepsilon}{6L}\right\}, t_2 - t_1 < \rho, \quad t_2^2 - t_1^2 < \eta, \quad t_2^3 - t_1^3 < \gamma, \quad (a^4+t^4) < \mu.$$

We have

$$|Tu(t_2) - Tu(t_1)| = \left| \begin{array}{l} \frac{a^2-b^2}{2!(a-b)}(t_2-t_1)\int_0^b sf(t,u(s))ds - \frac{a^3-b^3}{3!(a-b)}(t_2-t_1)\int_0^b f(t,u(s))ds + \\ \frac{1}{(a-b)}(t_2-t_1)\int_0^a \frac{(a-s)^3}{3!} f(t,u(s))ds - \\ \frac{1}{(a-b)}(t_2-t_1)\int_0^b \frac{(b-s)^3}{3!} f(t,u(s))ds - \frac{(t^2{}_2 - t^2{}_1)}{2!}\int_0^b sf(t,u(s))ds \\ + \frac{(t^3{}_2 - t^3{}_1)}{3!}\int_0^b f(t,u(s))ds - \int_0^a \frac{(a-s)^3}{3!} f(t,u(s))ds + \int_0^t \frac{(t-s)^3}{3!} f(t,u(s))ds \end{array} \right|$$

$$|Tu(t_2)-Tu(t_1)| \le \frac{(a^2-b^2)L}{(2!)(a-b)}(t_2-t_1)\left[\frac{b^2}{2}\right]+\frac{(a^3-b^3)L}{(3!)(a-b)}(t_2-t_1)[b]+\frac{L}{(a-b)}(t_2-t_1)\left[\frac{a^4}{4!}\right]+$$

$$\frac{L}{(a-b)}(t_2-t_1)\left[\frac{b^4}{4!}\right]+\frac{(t^2{}_2-t^2)L}{(2!)}\left[\frac{b^2}{2}\right]+\frac{(t^3{}_2-t^3{}_1)L}{(3!)}[b]+L\left[\frac{a^4}{4!}\right]+L\left[\frac{t^4}{4!}\right]$$

$$|Tu(t_2)-Tu(t_1)| \le \frac{(a^2-b^2)L}{(2!)(a-b)}\rho\left[\frac{b^2}{2}\right]+\frac{(a^3-b^3)L}{(3!)(a-b)}\rho[b]+\frac{L}{(a-b)}\rho\left[\frac{a^4}{4!}\right]+$$

$$\frac{L}{(a-b)}\rho\left[\frac{b^4}{4!}\right]+\frac{\eta L}{(2!)}\left[\frac{b^2}{2}\right]+\frac{\gamma L}{(3!)}[b]+\frac{L}{4!}\mu$$

$$\le \frac{L}{2!}\frac{(a^2-b^2)}{(a-b)}[\frac{b^2}{2}]\rho+\frac{L(a^3-b^3)}{2!(a-b)}[\frac{b}{3}]\rho+\frac{L(a^4+b^4)}{4!(a-b)}\rho+\frac{\eta L}{(2!)}\left[\frac{b^2}{2}\right]+\frac{\gamma L}{(3!)}[b]+\frac{L}{4!}\mu$$

$$\le \frac{\varepsilon}{6}+\frac{\varepsilon}{6}+\frac{\varepsilon}{6}+\frac{\varepsilon}{6}+\frac{\varepsilon}{6}+\frac{\varepsilon}{6} \le \varepsilon \qquad (3.7)$$

Thus $\overline{T(M)}$ is equicontinuous. The Arzela-Ascoli theorem implies that the operator T is completely continuous.

Theorem 3.2 Assume that $f:[0,1]xR \to R$ is continuous function, and there exist constants $0 < c_1 < \max(\frac{1}{Q}),\quad c_2 \succ o$,such that $|f(t,u(t))| \le c_1|u| + c_2$ for all $t \in [0,1]$. Then the boundary value problem (3.1) and (3.2) has a solution.

Proof: Following [6, 7,10], we will apply the nonlinear alternative theorem to prove that T has one fixed point.

Let $\Omega = \{u \in X; \|u\| \prec R\}$,be open subset of X , where

$$R \succ \left(6\left\{\frac{b^2}{2}Qc_1, bQc_1,+2Qc_1,\frac{b^2}{2}Qc_2, bQc_2,+2Qc_2\right\}\right).$$

$$Q < [\frac{a^2-b^2}{2!(a-b)}(a-t)+\frac{(a^2-t^2)}{2!},\left[\frac{a^3-b^3}{3!(a-b)}(a-t)+\frac{(a^3-t^3)}{3!}\right],$$

$$\frac{1}{(a-b)}(a-t)\left\{\frac{a^4+b^4}{4!}\right\},\ \frac{a^4+t^4}{4!}] \tag{3.8}$$

We suppose that there is a point $u \in \partial\Omega$ and $c_1 \in (0,1)$ such that $u = Tu$ so, for $u \in \partial\Omega$, we have:

$$|Tu(t)| = \left| \begin{array}{l} \frac{a^2-b^2}{2!(a-b)}(a-t)\int_0^b sf(t,u(s))ds - \frac{a^3-b^3}{3!(a-b)}(a-t)\int_0^b f(t,u(s))ds + \\ \frac{1}{(a-b)}(a-t)\int_0^a \frac{(a-s)^3}{3!} f(t,u(s))ds - \\ \frac{1}{(a-b)}(a-t)\int_0^b \frac{(b-s)^3}{3!} f(t,u(s))ds - \frac{(a^2-t^2)}{2!}\int_0^b sf(t,u(s))ds \\ + \frac{(a^3-t^3)}{3!}\int_0^b f(t,u(s))ds - \int_0^a \frac{(a-s)^3}{3!} f(t,u(s))ds + \int_0^t \frac{(t-s)^3}{3!} f(t,u(s))ds \end{array} \right| \tag{3.9}$$

$$\begin{aligned} |Tu(t)| \le {} & \frac{a^2-b^2}{2!(a-b)}(a-t)\int_0^b s|f(t,u(s))|ds + \frac{a^3-b^3}{3!(a-b)}(a-t)\int_0^b |f(t,u(s))|ds + \\ & \frac{1}{(a-b)}(a-t)\int_0^a \frac{(a-s)^3}{3!}|f(t,u(s))|ds + \\ & \frac{1}{(a-b)}(a-t)\int_0^b \frac{(b-s)^3}{3!}|f(t,u(s))|ds + \frac{(a^2-t^2)}{2!}\int_0^b s|f(t,u(s))|ds \\ & + \frac{(a^3-t^3)}{3!}\int_0^b |f(t,u(s))|ds + \int_0^a \frac{(a-s)^3}{3!}|f(t,u(s))|ds + \int_0^t \frac{(t-s)^3}{3!}|f(t,u(s))|ds \end{aligned} \tag{3.10}$$

$$\begin{aligned} |Tu(t)| \le {} & \frac{a^2-b^2}{2!(a-b)}(a-t)\int_0^b s(c_1|u(s)|+c_2)ds + \frac{a^3-b^3}{3!(a-b)}(a-t)\int_0^b (c_1|u(s)|+c_2)ds + \\ & \frac{1}{(a-b)}(a-t)\int_0^a \frac{(a-s)^3}{3!}(c_1|u(s)|+c_2)ds + \\ & \frac{1}{(a-b)}(a-t)\int_0^b \frac{(b-s)^3}{3!}(c_1|u(s)|+c_2)ds + \frac{(a^2-t^2)}{2!}\int_0^b s(c_1|u(s)|+c_2)ds \\ & + \frac{(a^3-t^3)}{3!}\int_0^b (c_1|u(s)|+c_2)ds + \int_0^a \frac{(a-s)^3}{3!}(c_1|u(s)|+c_2)ds + \int_0^t \frac{(t-s)^3}{3!}(c_1|u(s)|+c_2)ds \end{aligned} \tag{3.11}$$

$$|Tu(t)| \le \frac{a^2-b^2}{2!(a-b)}(a-t)\left\{\frac{b^2}{2}(c_1|u(s)|+c_2)\right\}+\frac{a^3-b^3}{3!(a-b)}(a-t)\{b(c_1|u(s)|+c_2)\}+$$
$$\frac{1}{(a-b)}(a-t)\left\{\frac{(a)^4}{4!}(c_1|u(s)|+c_2)\right\}+$$
$$\frac{1}{(a-b)}(a-t)\left\{\frac{(b)^4}{4!}(c_1|u(s)|+c_2)\right\}+\frac{(a^2-t^2)}{2!}\left\{\frac{b^2}{2}(c_1|u(s)|+c_2)\right\}$$
$$+\frac{(a^3-t^3)}{3!}\{b(c_1|u(s)|+c_2)\}+\left\{\frac{(a)^4}{4!}(c_1|u(s)|+c_2)\right\}+\frac{(t)^4}{4!}(c_1|u(s)|+c_2)$$

$$\begin{aligned}|Tu(t)| \le &\left[\frac{a^2-b^2}{2!(a-b)}(a-t)+\frac{(a^2-t^2)}{2!}\right]\left\{\frac{b^2}{2}(c_1|u(s)|+c_2)\right\}+\\ &\left[\frac{a^3-b^3}{3!(a-b)}(a-t)+\frac{(a^3-t^3)}{3!}\right]\{b(c_1|u(s)|+c_2)\}+\\ &\frac{1}{(a-b)}(a-t)\left\{\frac{a^4+b^4}{4!}(c_1|u(s)|+c_2)\right\}+\left\{\frac{a^4+t^4}{4!}(c_1|u(s)|+c_2)\right\}\end{aligned} \tag{3.12}$$

$$|Tu(t)| \le Q\left\{\frac{b^2}{2}(c_1|u(s)|+c_2)\right\}+$$
$$Q\{b(c_1|u(s)|+c_2)\}+Q\{(c_1|u(s)|+c_2)\}+\{Q(c_1|u(s)|+c_2)\}$$

$$\begin{aligned}|Tu(t)| \le &Q\left\{\frac{b^2}{2}(c_1|u(s)|)\right\}+Q\frac{b^2}{2}c_2+Q\{b(c_1|u(s)|)\}+Qbc_2+2Q\{(c_1|u(s)|)\}+2Qc_2\\ &\prec \frac{R}{6}+\frac{R}{6}+\frac{R}{6}+\frac{R}{6}+\frac{R}{6}+\frac{R}{6}=\mathrm{R},\end{aligned} \tag{3.13}$$

which implies that $\|T\| \ne R = \|u\|$, that is a contraction. Then the nonlinear alternative theorem implies that T has a fixed point $u \in \overline{\Omega}$, that is , problem(3.1) and (3.2) has a solution $u \in \overline{\Omega}$.

Finally, we give an example to illustrate the results obtained in this paper.

Example: For the boundary value problem

$$u^3(t) = \frac{u}{u^2+8} \tag{3.14}$$

By using Eq. (3.1) with boundary condition (3.2) and applying theorem 3.2, we have

$$c_1 = 1 \prec \max \frac{1}{Q} \cdot$$

We come to the conclusion that problem (3.14) has a solution.

Conclusion

In this paper, the existence and nonexistence of positive solutions for nonlinear higher order boundary value problem were studied by using the fixed point theory. We conclude that the mentioned 4^{th} order differential equation has at least one positive solution.

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حلول موجبه لمسألة القيم الحدية ثنائية النقطة الشاذة من الرتبة الرابعة

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ملخص البحث. تبحث هذه الورقة وجود أو عدم وجود حلول موجبه للمعادلات التفاضلية الغير خطية من الرتب العليا. ويمكن تحقيق ذلك باستخدام نظريات مناسبة عن طريق نظرية النقطة الثابتة لدراسة خصائص المعادلة التفاضلية المستمرة غير الخطية في فضاء باناخ على فترة محدودة ومغلقة. وتبين لنا أنه يوجد حل موجب واحد على الأقل للمعادلة التفاضلية من الرتبة الرابعة.