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Multifractal character in the charge distribution of Projectile fragments

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Abstract. The nuclear fragmentation data obtained from ^{24}Mg -Em interaction at 4.5 AGeV is analyzed to perform dynamical fluctuation and hence multifractal characters using the modified G_q moment method. The calculated values of the generalised fractal dimension D_q, are found to decrease with the increasing order of moments, q. The analysis clearly shows the existence of dynamical fluctuation thereby indicating multifractal behavior in the fragments charge distribution.

Keywords: Nuclear fragmentation, generalized fractal dimension, multifractality

1. Introduction

The self-similarity observed in the power law dependence of scaled factorial moments reveals a connection between intermittency and fractality. This self-similar properties were first developed by Bialas and Peschanski [1-2] in the study of multiplicity fluctuation in particle production at high energy. In multifractal approach, it has been suggested that the nuclear interactions can be treated as geometrical objects with non-integer dimensions. The study of chaotic system and intermittent behavior in turbulent fluids has been performed using the fractal dimension. Out of various methods that have been proposed to investigate the fractal structure, Hwa [3] was the first to provide the idea of the multifractal moments G_q to study the multifractality and self-similarity in multiparticle production. If the particle process exhibits self-similar behavior, then a modified form of G_q moment in terms of step function shows the remarkable power law dependence on phase space bin size.

In this article an attempt has been made to study the characteristics of dynamical fluctuation and hence multifractal nature in the charge distribution of projectile fragments from ^{24}Mg -Em at 4.5 AGeV. The experimental data for this work have been obtained by NIKFI BR-2 nuclear emulsion pellicles that are irradiated parallel to their lengths by a 4.5A GeV ^{24}Mg beam from the JINR synchrophasotron at Dubna. All total 650 minimum biased Mg-Em interactions are considered for the present work. The details of the experimental work has been given in some previous work [4-5].

2. Mathematical formalism

The particle number density in each rapidity bin depends on whether the resolution of the binning is of the order of or better than the rapidity separation between neighboring particles [6]. It has been found that if the resolution is of the order of the average separation of two neighboring particles in the phase space, then the binning of the phase space with that resolution may result in some empty bins. Considering the empty bins in the distribution are analogous to the holes then the set of non-empty bins would form a fractal set [3].

By introducing a step function to suppress the low multiplicity events for which the statistical fluctuation is large, Hwa and Pan [7] proposed a modified G_q moment to investigate the fractal properties of the emission spectra of different charged secondaries. With this concept of fractality, the fractal moments G_q have been defined to evaluate parameters which characterize the fractal properties. In this investigation, the generalized moments G_q is first estimated using the relation

$$G_q = \sum_{m=1}^{M} \left(\frac{n_m}{N_{eV}} \right)^q \; □ \, (n_m - q)$$

Here □$(n_m - q)$ is a step function such that □$(n_m - q) = 1$ for $n_m > q$, and □$(n_m - q) = 0$ for $n_m < q$. For an ensemble of events, the averaging is done as

$$< G_q > = \frac{1}{N_{eV}} \sum G_q$$

If the charge distributions have fractal structure, then according to the theory, for the charge distribution of projectile fragments, the G_q moment should also follow a power law i.e.

$$< G_q > \infty \; (\delta s)^{\tau_q}$$

where τ_q is the fractal index or mass exponent. From the linear dependence of ln< G_q > on ln δs, τ_q can be calculated as

$$\tau_q = \frac{\Delta \ln \langle G_q \rangle}{\Delta \ln (\delta s)} .$$

3. Results and discussion

Now to calculate the statistical contribution to $< G_q >$, equal number of events are also generated by random number generator with charge of the PF lying between 1 to 12 for Mg projectile beams.

$< G_q >^{stat}$ is then calculated for uncorrelated projectile fragments in randomly generated events. The dynamical part of $< G_q >$ can be determined from the relation [8]

$$< G_q >^{dyn} = \frac{< G_q >}{< G_q >^{stat}} (\delta s)^{q-1}$$

Variation of ln $< G_q >$ with -ln δs is plotted in Fig. 1. A significant difference could easily be seen between the experimental and random data for the projectile beam.

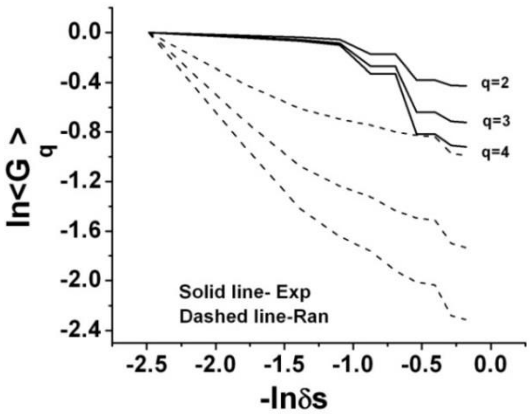

Fig. (1). ln< G_q > variation with -ln δs

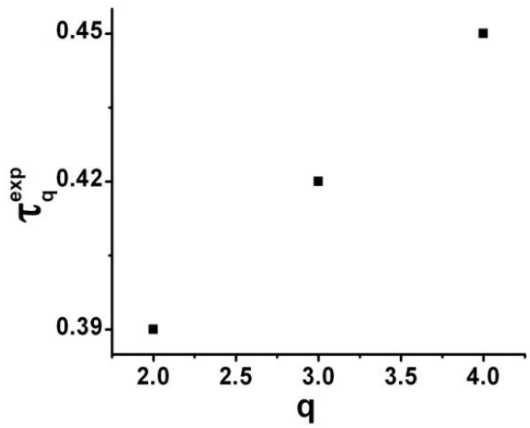

Fig .(2). τ(exp) variation with q

The various exponents obtained from the experimental data are plotted in Fig. 2. The exponents are found increase with the increase of the order of the moments. Since the exponent τ_q^{exp} as obtained above, are estimated from the experimental data set, it contains statistical component also. The dynamical component of exponent τ_q is then estimated using the relation

$$\tau_q^{dyn} = \tau_q^{exp} - \tau_q^{st} + q\text{-}1$$

where τ_q^{st} represents the slopes of randomly generated data obtained from ln< G_q > vs ln δs plot. In Fig. 3 we have shown the variation of these mass exponents namely with q. It is clearly seen from this plot that, τ_q^{dyn} significantly varies from q-1 indicating the presence of dynamical fluctuation.

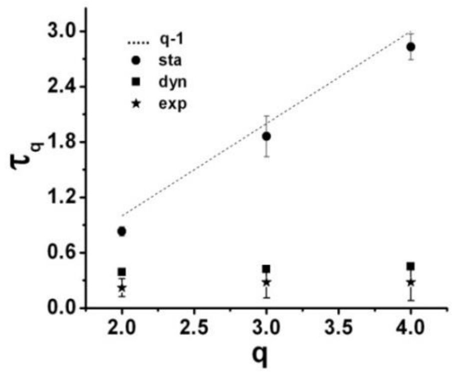

Fig .(3). τ_q^{dyn} deviation from q-1

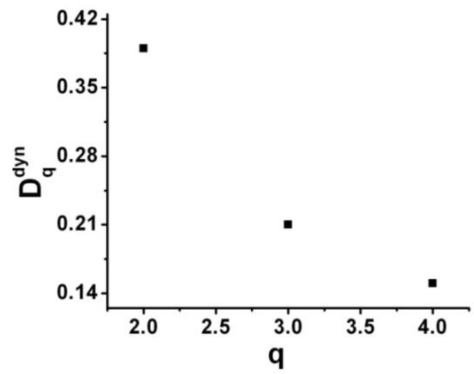

Fig. (4). D_q^{dyn} variation with q

Again τ_q is related to the generalized dimension D_q, introduced by Hentschal & Procaccia [9] through the relation

$$D_q^{dyn} = \frac{\tau_q^{dyn}}{q-1}$$

If we plot D_q^{dyn} against q it is found that D_q^{dyn} gradually decreases with the increase of q indicating the presence of multifractality. The value of generalized dimensions D_q for various values of q have been estimated for the charge distribution of projectile in 4.5 AGeV ^{24}Mg nuclei with emulsion target , and their variations with different order of moments q = 2 - 4 are shown in figure 4. It is observed from the figure that the generalized dimensions D_q decreases linearly with q for the analysis, showing an evidence of multictral nature in the charge distribution of projectile fragments.

Table (1). The values of different mass exponents with q

q	$\tau_q^{\exp}$	τ_q^{dyn}	q-1- τ_q^{dyn}	D_q^{dyn}
2	0.21 ± 0.099	0.40 ± 0.015	0.60	0.40
3	0.28 ± 0.017	0.43 ± 0.047	1.57	0.22
4	0.29 ± 0.092	0.46 ± 0.149	2.54	0.16

The values of τ_q for experimental data points, τ_q^{dyn} , q – 1 – τ_q^{dyn} and D_q for different orders of moment are listed in table 1. It is observed from this table that the value of τ_q^{dyn} deviates from q – 1 and this deviation is more as we go to the higher order of moments. This indicates that G_q contains information about multifractal behavior of fluctuations.

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