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A New Decision Model Based on the Value of Fuzzy Numbers

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Abstract. In this paper, we discuss the arithmetic mean operation of Trapezoidal Fuzzy Numbers (TrFNs). We also explain the value of both Trapezoidal and Triangular Fuzzy Numbers (TFNs). Then we propose new multi criteria fuzzy decision making model based on the value of fuzzy numbers which in turn will be very helpful in developing fuzzy expert systems. Here arithmetic mean operation of fuzzy numbers is used for compiling experts' judgments.

***Key words*:** Trapezoidal and Triangular fuzzy numbers, Arithmetic mean, Value of fuzzy numbers, Expert systems.

Introduction

Most of the real life problems are complex in nature because of the indistinctness and impreciseness of the available data. In 1970 Bellman and Zadeh proposed the concept of fuzzy sets and fuzzy models to effectively handle these imprecise data which help us to avoid information loss through computing with words. To solve such real world problems, we can develop fuzzy expert systems by seeking the help of experts who have knowledge in that particular area. There may be many factors that influence a certain problem. While developing expert system, one has to rank these factors based on the experts' judgements. Usually experts' opinion is obtained as linguistic variables which can easily be converted into fuzzy numbers. For arriving at conclusions, we need to compile the experts' judgements. Subsequently, we use some ranking methods to find the order of these factors.

Ranking of fuzzy numbers plays a very significant role in linguistic multi-criteria decision making problems. Several fuzzy ranking methods have been proposed since 1976. The linguistic terms are represented quantitatively using fuzzy sets and then fuzzy optimal alternative is calculated which gives the relative merit of each alternative. S. Abbasbandy and T.Hajjari [2] in 2009 proposed a new method based on the left and right spreads at some α – levels and defined magnitude of fuzzy numbers. Ranking is done based on this magnitude. S. Abbasbandy and B. Asady [1] in 2006 proposed sign distance method by considering a fuzzy origin and then calculating distance with respect to the origin. If $u \in X$ and u_0 is the origin, then distance is defined as

$$D_p(u, u_0) = \left[\int_0^1 \left(\left| \underline{u}(r) \right|^p + \left| \overline{u}(r) \right|^p \right) dr \right]^{1/p}, \qquad p \geq 1$$

where $\underline{u}(r) = x_0 - \sigma + \sigma r$ and $\overline{u}(r) = y_0 + \beta - \beta r$ are the parametric form of the fuzzy number $u = (x_0, y_0, \sigma, \beta)$ with two defuzzifiers x_0 , y_0 and left fuzziness $\sigma > 0$ and right fuzziness $\beta > 0$.

F. Choobineh and Huishen Li [11] in 1993 proposed a new index for ranking without taking account of normality or convexity of fuzzy numbers. The new index for the fuzzy set A is defined as

$$R(a) = \frac{1}{2}\left[h_A - \frac{D\left(\mu_A, \mu_{U_A}\right) - D\left(\mu_A, \mu_{L_A}\right)}{d - a} \right] \; ; a \in [0,1], d \in [0,1],$$

where h_A is the height of the fuzzy set A and μ_{U_A} and μ_{L_A} are the membership functions for the crisp barriers of the fuzzy set A. Ronald R.Yager [14] in 1981 proposed a ranking method using a function which is the integral of the mean of the level sets of the fuzzy subsets. The function F is defined from the subsets of unit interval I into I; for the fuzzy set A of I, as in the following formula

$$F(A) = \int_0^{\alpha_{max}} M \, d\alpha,$$

where α_{max} is the maximum membership grade and M is the mean value of the members of an ordinary subset of the unit interval I.

Bass and H. Kwakernaak [6] proposed a method in 1977 consisting of computing weighted final ratings for each alternative and comparing the final weighted rating. J.F. Baldwin and NCF Guild [3] in 1979 improved the procedure proposed by Bass and Kwakernaak. They introduced a new method for pair wise comparison of all the alternatives instead of ranking them in the set of alternatives. This new method is helpful to determine how much one factor is greater than the other. Chung-Tsen Tsao 2002 [12] proposed a ranking method with the area between the centroid point and original point. J. Yao and K.Wu [15] in 2000 defined signed distance for ordering fuzzy numbers using decomposition principle. Signed distance between the fuzzy sets A and B is defined as

$$d(A,B) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha) - B_L(\alpha) - B_R(\alpha)]\, d\alpha.$$

Where $[A_L, A_R]$ and $[B_L, B_R]$ are the α-cuts of A and B respectively.

Lee and Li [13] in 1988 proposed a method based on the probability measures such as mean and standard deviation. Ching –Hsue Cheng in 1998 [10] proposed a method for ranking more than two fuzzy numbers simultaneously without considering the normality of fuzzy numbers. Ranking function is defined as the distance between centroid point and the original point. Based on the ranking function, ranking is determined. To improve Lee and Li's method, C.H Cheng proposed a ranking method also based on the coefficient of variation.

The Arithmetic mean operation of TrFNs is introduced here, which will be more advantageous in ranking procedure, as this method is easier to compile the variety of experts' judgements. Then a new ranking method based on the values of the fuzzy numbers is explained. This paper is organized as follows: Section 1 presents the preliminaries; Section 2 depicts the method of finding Arithmetic Mean of TrFNs; Section 3 describes the value of fuzzy numbers; Section 4 explains the new decision making model with illustrations and Section 5 concludes the work.

1. Preliminaries

Definition 1.1

A fuzzy set A in a universe of discourse X is defined as the set of pairs,

$A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x): X \to [0,1]$ is called the membership value of $x \in X$ in the fuzzy set A.

Definition 1.2 [4]

The α-cut, $\alpha \in (0,1]$ of a fuzzy number A is a crisp set defined as $A(\alpha) = \{x \epsilon R : A(x) \geq \alpha\}$. Every A_α is a closed interval of the form $[A_L(\alpha), A_U(\alpha)]$.

Definition 1.3

For a fuzzy set A of X, the support of A, denoted by $supp(A)$ is the crisp subset of X which contains elements having nonzero membership grades in A.

i.e. , $supp(A) = \{x \in X: \mu_A(x) > 0\}$.

Definition 1.4 [4]

A fuzzy number A is a fuzzy subset of the real line; $A: R \rightarrow [0,1]$ satisfying the following properties:

(i) A is normal (i.e. there exists $x_0 \in R$ such that $A(x_0) = 1$);

(ii) A is fuzzy convex ;

(iii) A is upper semi continuous on R. ie; $\forall \varepsilon > 0$, $\exists\, \delta > 0$ such that $A(x) - A(x_0) < \varepsilon$ whenever $|x - x_0| < \delta$;

(iv) The closure , $cl(supp(A))$ is compact.

Definition 1.5 [4]

The set $A(\alpha)$ is a convex set if $x, y \in A(\alpha) \Rightarrow \lambda x + (1-\lambda) y \in A(\alpha)$ where $\lambda \in [0,1]$ and $\alpha \in [0,1]$.

Definition 1.6 [5]

A Trapezoidal fuzzy number denoted by A is defined as (l, m, n, u) where the membership function is given by

$$\mu_A(x) = \begin{cases} 0, & x \le l \\ \dfrac{x-l}{m-l}, & l \le x \le m \\ 1, & m \le x \le n \\ \dfrac{u-x}{u-n}, & n \le x \le u \\ 0, & x \ge u. \end{cases}$$

Definition 1.7 [4]

The value of a fuzzy number A is denoted and defined as $val(A) = \int_0^1 \alpha\left(A_U(\alpha) + A_L(\alpha)\right) d\alpha$.

2. Arithmetic Mean Operation of Trapezoidal Fuzzy Numbers (TrFNs)

Consider the Trapezoidal Fuzzy Numbers:

$A_1 = (a_1, a_2, a_3, a_4)$, $A_2 = (b_1, b_2, b_3, b_4), \dots,$ $A_n = (n_1, n_2, n_3, n_4)$ with membership functions,

$$\mu_{A_1}(x_1) = \begin{cases} 0\ , & x_1 \le a_1 \\ \dfrac{x_1 - a_1}{a_2 - a_1}, & a_1 \le x_1 \le a_2 \\ 1\ , & a_2 \le x_1 \le a_3 \\ \dfrac{a_4 - x_1}{a_4 - a_3}, & a_3 \le x_1 \le a_4 \\ 0\ , & x_1 \ge a_4 \end{cases}$$

$$\mu_{A_2}(x_2) = \begin{cases} 0\ , & x_2 \le b_1 \\ \dfrac{x_2 - b_1}{b_2 - b_1}, & b_1 \le x_2 \le b_2 \\ 1\ , & b_2 \le x_2 \le b_3 \\ \dfrac{b_4 - x_2}{b_4 - b_3}, & b_3 \le x_2 \le b_4 \\ 0\ , & x_2 \ge b_4 \end{cases}$$

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$$\mu_{A_n}(x_n) = \begin{cases} 0\ , & x_n \le n_1 \\ \dfrac{x_n - n_1}{n_2 - n_1}, & n_1 \le x_n \le n_2 \\ 1\ , & n_2 \le x_n \le n_3 \\ \dfrac{n_4 - x_n}{n_4 - n_3}, & n_3 \le x_n \le n_4 \\ 0\ , & x_n \ge n_4 \end{cases}$$

Or,

$$\mu_{A_1}(x_1) = max\left[min\left(\frac{x_1 - a_1}{a_2 - a_1}, 1, \frac{a_4 - x_1}{a_4 - a_3}\right), 0\right]$$

$$\mu_{A_2}(x_2) = max\left[min\left(\frac{x_2 - b_1}{b_2 - b_1}, 1, \frac{b_4 - x_2}{b_4 - b_3}\right), 0\right]$$

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$$\mu_{A_n}(x_n) = max\left[min\left(\frac{x_n - n_1}{n_2 - n_1}, 1, \frac{n_4 - x_n}{n_4 - n_3}\right), 0\right]$$

The α- cuts of these fuzzy numbers are given by:

$$A_1(\alpha) = [A_{1L}(\alpha), A_{1U}(\alpha)] = [a_1 + \alpha(a_2 - a_1),\ a_4 - \alpha(a_4 - a_3)]$$

$$A_2(\alpha) = [A_{2L}(\alpha), A_{2U}(\alpha)] = [b_1 + \alpha(b_2 - b_1),\ b_4 - \alpha(b_4 - b_3)]$$

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$$A_n(\alpha) = [A_{nL}(\alpha), A_{nU}(\alpha)] = [n_1 + \alpha(n_2 - n_1),\ \ n_4 - \alpha(n_4 - n_3)]$$

Then we define the Arithmetic Mean of these fuzzy numbers as follows:

Let $A_V = \frac{A_1 + A_2 + \cdots + A_n}{n} = \left(\frac{a_1+b_1+\cdots+n_1}{n}, \frac{a_2+b_2+\cdots+n_2}{n}, \frac{a_3+b_3+\cdots+n_3}{n}, \frac{a_4+b_4+\cdots+n_4}{n}\right)$ with membership function,

$$\mu_{A_V}(X) = \begin{cases} sup\left[min\left(\frac{x_1-a_1}{a_2-a_1}, \frac{x_2-b_1}{b_2-b_1}, \dots, \frac{x_n-n_1}{n_2-n_1}\right): \frac{x_1+x_2+\cdots+x_n}{n} = X\right] \\ \quad if \quad a_1 \le x_1 \le a_2, b_1 \le x_2 \le b_2, \dots, n_1 \le x_n \le n_2 \\ 1 \quad if \quad a_2 \le x_1 \le a_3, b_2 \le x_2 \le b_3, \dots, n_2 \le x_n \le n_3 \\ sup\left[min\left(\frac{a_4-x_1}{a_4-a_3}, \frac{b_4-x_2}{b_4-b_3}, \dots, \frac{n_4-x_n}{n_4-n_3}, \right): \frac{x_1+x_2+\cdots+x_n}{n} = X\right] \\ \quad if \quad a_3 \le x_1 \le a_4, b_3 \le x_2 \le b_4, \dots, n_3 \le x_n \le n_4 \end{cases}$$

That is

$$\mu_{A_V}(X) = \begin{cases} \frac{X - \left(\frac{a_1+b_1+\cdots+n_1}{n}\right)}{\left(\frac{a_2+b_2+\cdots+n_2}{n}\right) - \left(\frac{a_1+b_1+\cdots+n_1}{n}\right)} & if\ \left(\frac{a_1+b_1+\cdots+n_1}{n}\right) \le X \le \left(\frac{a_2+b_2+\cdots+n_2}{n}\right) \\ 1 & if\ \left(\frac{a_2+b_2+\cdots+n_2}{n}\right) \le X \le \left(\frac{a_3+b_3+\cdots+n_3}{n}\right) \\ \frac{\left(\frac{a_4+b_4+\cdots+n_4}{n}\right) - X}{\left(\frac{a_4+b_4+\cdots+n_4}{n}\right) - \left(\frac{a_3+b_3+\cdots+n_3}{n}\right)} & if\ \left(\frac{a_3+b_3+\cdots+n_3}{n}\right) \le X \le \left(\frac{a_4+b_4+\cdots+n_4}{n}\right) \\ 0 & otherwise. \end{cases}$$

The α- cut of A_V is given as:

$$\begin{aligned} A_V(\alpha) &= [A_{VL}(\alpha),\ A_{VU}(\alpha)] \\ &= \left[\left\{\left(\frac{a_1+b_1+\cdots+n_1}{n}\right) + \alpha\left(\frac{a_2+b_2+\cdots+n_2}{n} - \frac{a_1+b_1+\cdots+n_1}{n}\right)\right\}, \right. \\ &\quad \left.\left\{\left(\frac{a_4+b_4+\cdots+n_4}{n}\right) - \alpha\left(\frac{a_4+b_4+\cdots+n_4}{n} - \frac{a_3+b_3+\cdots+n_3}{n}\right)\right\}\right] \end{aligned}$$

Note: Similarly we can define arithmetic mean operation for triangular fuzzy numbers.

3. Value of Fuzzy Numbers

Proposition 3. 1: The value of a Trapezoidal fuzzy number $A = (a, b, c, d)$ is given by

$$val(A) = \frac{a}{6} + \frac{b}{3} + \frac{c}{3} + \frac{d}{6}$$

Proof:

The $\alpha - cut$ of the Trapezoidal fuzzy number $A = (a, b, c, d)$ is given by

$$A(\alpha) = [A_L(\alpha), A_U(\alpha)] = [a + \alpha(b - a), \; d - \alpha(d - c)]$$

Then the value of the fuzzy number $A = (a, b, c, d)$ is denoted as $val(A)$ and is defined by

$$val(A) = \int_0^1 \alpha \, [A_U(\alpha) + A_L(\alpha)] d\alpha$$

$$= \int_0^1 \alpha \, [d - \alpha(d - c) + a + \alpha(b - a)] d\alpha$$

$$= \frac{a}{6} + \frac{b}{3} + \frac{c}{3} + \frac{d}{6}$$

Proposition 3. 2: The value of a Triangular fuzzy number $B = (a, b, c)$ is given by

$$val(B) = \frac{a}{6} + \frac{2b}{3} + \frac{c}{6}$$

Proof:

The $\alpha - cut$ of the Triangular fuzzy number $B = (a, b, c)$ is given by

$$B(\alpha) = [B_L(\alpha), B_U(\alpha)] = [a + \alpha(b - a), \; c + \alpha(b - c)]$$

$$val(B) = \int_0^1 \alpha \, [A_U(\alpha) + A_L(\alpha)] d\alpha$$

$$= \int_0^1 \alpha \, [c + \alpha(b - c) + a + \alpha(b - a)] d\alpha$$

$$= \frac{a}{6} + \frac{2b}{3} + \frac{c}{6}$$

4. New Decision Making Model [7],[8],[9]

The significant step in decision making model is identifying factors and sub factors which are specific to the problem. Then the process involves obtaining an appropriate set of linguistic variables and associated fuzzy numbers in order to rank these factors. These fuzzy numbers can be shown by membership functions. The selection of linguistic variables is chiefly a combination of knowledge elicitation and data preparation. We collect experts' judgments through questionnaires which are obtained as linguistic variables. These linguistic terms are fuzzified using the associated fuzzy numbers. We have to compile experts' judgements to have a group agreement of all experts. This step is done using arithmetic mean operation of fuzzy numbers which in turn result a single judgment fuzzy number for each of the factors. Then ranking can be done based on the values of these fuzzy numbers.

Illustration

Consider a problem with n factors. Then judgement matrix for the i^{th} factor (a_{ik}) can be formed where $i = 1,2,..,n$ and k is the number of experts.

We construct the judgement matrix of the form:

$$(a_{ik}) = \begin{bmatrix} l_{i1} & m_{i1} & n_{i1} & u_{i1} \\ l_{i2} & m_{i2} & n_{i2} & u_{i2} \\ \cdots & \cdots & \cdots & \cdots \\ l_{ik} & m_{ik} & n_{ik} & u_{ik} \end{bmatrix}$$ where $1 \leq i \leq n$ and k is the number of experts.

Arithmetic mean operation is used for aggregation of the judgments of various experts.

$$A_i = (l_i, m_i, n_i, u_i) = \left(\frac{l_{i1}+\cdots+l_{ik}}{k}, \frac{m_{i1}+\cdots+m_{ik}}{k}, \frac{n_{i1}+\cdots+n_{ik}}{k}, \frac{u_{i1}+\cdots+u_{ik}}{k}\right)$$ where $1 \leq i \leq n$ and k is the number of experts.

The value of the single judgment fuzzy number A_i of the i^{th} factor is given by

$$val(A_i) = \frac{l_i}{6} + \frac{m_i}{3} + \frac{n_i}{3} + \frac{u_i}{6}$$

Similarly we find the values corresponding to all factors. Based on these values we determine the ranking of A_i and A_j as follows:

(i) $val(A_i) > val(A_j) \Rightarrow A_i > A_j$

(ii) $val(A_i) < val(A_j) \Rightarrow A_i < A_j$

(iii) $val(A_i) = val(A_j) \Rightarrow A_i \sim A_j$

This method is used in the case of triangular fuzzy numbers also.

Remark 4.1:

(i) $val(A_1 + A_2 + \cdots + A_n) = val(A_1) + val(A_2) + \cdots + val(A_n)$

(ii) $val\left(\frac{A_1+A_2+\cdots+A_n}{n}\right) = \frac{val(A_1)+val(A_2)+\cdots+val(A_n)}{n}$

Example 4.1:

Consider the following set of fuzzy numbers[2]:

Set 1: $A_1 = (0.5, 0.1, 0.5)$, $A_2 = (0.7, 0.3, 0.3)$, $A_3 = (0.9, 0.5, 0.1)$

Set 2: $A_1 = (0.4, 0.7, 0.1, 0.2)$, $A_2 = (0.7, 0.4, 0.2)$, $A_3 = (0.7, 0.2, 0.2)$

Set 3: $A_1 = (0.5, 0.2, 0.2),\ A_2 = (0.5, 0.8, 0.2, 0.1),\ A_3 = (0.5, 0.2, 0.4)$

Set 4: $A_1 = (0.4, 0.7, 0.4, 0.1),\ A_2 = (0.5, 0.3, 0.4),\ A_3 = (0.6, 0.5, 0.2)$

The ranking based on the proposed method is shown as the following table:

Ranking based on the proposed method:				
Fuzzy Number	Set 1	Set 2	Set 3	Set 4
A_1	0.2333	0.3666	0.2499	0.45
A_2	0.3667	0.4167	0.4334	0.35
A_3	0.5	0.2833	0.2833	0.4666
Results	$A_1 < A_2 < A_3$	$A_3 < A_1 < A_2$	$A_1 < A_3 < A_2$	$A_2 < A_1 < A_3$
Comparison of ranking of the above sets of fuzzy numbers based on some other methods[2]:				
Method/Authors	Set 1	Set 2	Set 3	Set 4
S. Abbasbandy (Magnitude)	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_3 < A_2$	$A_2 < A_1 < A_3$
S. Abbasbandy (Sign distance method p=1)	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_3 < A_2$	$A_2 < A_1 \sim A_3$
S. Abbasbandy (Sign distance method p=2)	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_3 < A_2$	$A_2 < A_1 < A_3$
Choobineh and Li	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$
Yager	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$
Chen	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$
Baldwin and Guild	$A_1 < A_2 < A_3$	$A_1 \sim A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 \sim A_2 < A_3$
Chu and Tsao	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_3 < A_2$	$A_1 < A_3 < A_2$
Yao and Wu	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_3 < A_2$	$A_2 < A_1 \sim A_3$
Cheng distance	$A_1 < A_2 < A_3$	$A_1 < A_2 < A_3$	$A_1 < A_3 < A_2$	$A_1 < A_3 < A_2$
Cheng CV uniform distribution	$A_2 < A_3 < A_1$	$A_3 < A_2 < A_1$	$A_1 < A_3 < A_2$	$A_2 < A_3 < A_1$
Cheng CV proportional distribution	$A_2 < A_3 < A_1$	$A_3 < A_2 < A_1$	$A_1 < A_3 < A_2$	$A_2 < A_3 < A_1$

Furthermore, the images of the above of fuzzy numbers are considered as the following.

Set 1: $-A_1 = (-0.5, -0.1, -0.5),\ A_2 = (-0.3, -0.3, -0.7),\ A_3 = (-0.1, -0.5, -\ 0.9)$

$$val(A_1) = -0.2333,\ val(A_2) = -0.3667,\ val(A_3) = -0.5$$

The ranking order is $-A_3 < -A_2 < -A_1 \Rightarrow A_1 < A_2 < A_3$

Set 2: $-A_1 = (-0.2, -0.1, -0.7, -0.4),\ A_2 = (-0.2, -\ 0.4, -\ 0.7),\ A_3 = (-0.2, -0.2, -0.7)$

$$val(A_1) = -0.3666,\ val(A_2) = -0.4167,\ val(A_3) = -0.2833$$

The ranking order is $-A_2 < -A_1 < -A_3 \Rightarrow A_3 < A_1 < A_2$

Similarly we can check the images for Set 3 and Set 4 also. This shows our new method has consistency in ranking fuzzy numbers and their images.

Conclusion

We have defined the values of different fuzzy numbers. Based on these values we proposed a simple ranking method for trapezoidal and triangular fuzzy numbers. The proposed method can successfully rank fuzzy numbers and their images. This new method does not require much computational effort in the ranking procedure and it depends only on the experts' judgments. We have given some comparative results also.

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