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On the origin of a longitudinal force

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Abstract: Inertial effects arising from the electron mass are investigated. A longitudinal force is generated when a large current is passed in a wire. Because of this force, an additional magnetic field due to the electron mass is generated along the wire axis. When a huge current is applied to the wire, the wire explodes giving off its magnetic energy as heat and light. Massive electrodynamics is studied which allows the presence of magnetic charges (monopoles). The exploding force on a wire could also result from disintegrating the magnetic monopoles.

Key Words: Inertia, Kinetic self-inductance, Quantum inductance, Quantum magnetic flux

1 Introduction

Newton's second law states that an object has intrinsic resistance to a change in its motion. The tendency to which the object resists this change is termed inertia. It is found to be equal to the mass the object has. The resistance of an object to increase or decrease its velocity is the same. However, this inertia is reflected in many ways in any change in body characteristics. If an object is charged there is an additional resistance (inertia) due to its charge as well. The passage of a charge gives rise to an electric current. Self-inductance is the degree of the resistance of the coil (inductor) for increasing or decreasing the current passing on it. Since electrons have mass and charge, they experience two inertias. For instance, the inductor (coil) has an additional inertial self-inductance besides its magnetic one. Hence, one anticipates that these two effects are interrelated and manifested in other aspects.

In Mach idealogy, inertia is not an intrinsic property of the object but is a manifestation of how the object is influenced by the external world. Accordingly, an object in an otherwise empty universe has vanishing inertia (mass)[1]. This is so since there is no force acting on a single object, and owing to Newton's second law that $F = ma = 0$, which is meaningful when $m = 0$. This is so since a body moving with constant velocity needs a reference frame, that does not exist, making this assumption absurd. Hence, inertia is affected by the amount of matter appearing outside the object.

To obtain the complete physical quantities connected with the object's charge and mass inertias, we resort to the analogy connecting electromagnetism with mechanics. Inertial effects are not considered in the framework of Maxwell's electrodynamics, as they are connected with quantum mechanics. We aim here to explore their origin and consequences.

The inertial motion of the electron is found to give rise to an inertial magnetic field that acts along the wire length. This magnetic field is shown to give rise to a longitudinal (tension) force along the wire. It is pertinent to mention that there is a component of the magnetic field (TE waves) inside a waveguide. It is found that this motion is equivalent to that of a massive photon. The massive electrodynamics was considered by Proca and de Broglie[2, 3]. To derive new massive electrodynamics we employ quaternions. This massive electrodynamics is associated with breaking gauge invariance. We will show here that massive electrodynamics can be tackled if the Lorenz gauge condition is relaxed.

2 Inertial effects in electromagnetism

Let us now consider the motion of electrons inside a conductor. The current passing through this conductor is given by

$$I = nevA \quad , \tag{1}$$

where A, v, e and n are the cross-sectional area of the conductor, electron velocity, electron charge and the number density of the electrons[2]. The kinetic energy of the total mass

[2] The number density, $n = \frac{N_A}{M}\ \rho_m$, where ρ_m is the mass density of the material, M is its atomic weight, and N_A is the Avogadro's number.

of electrons inside the conductor is given by

$$E_K = \frac{1}{2} \ M v^2 \ , \tag{2}$$

where M is the total mass of the electrons inside the conductor. It is given by

$$M = mnA\ell \ , \tag{3}$$

where m is the mass of a single electron. Substituting eqs.(1) and (3) in eq.(2) yields

$$E_K = \frac{1}{2} \ \left(\frac{m\ell}{ne^2 A}\right) \ I^2 \ , \tag{4}$$

which can be compared with the magnetic energy contained inside an inductor (coil) arising from the motion of electron's charge, *viz*.,

$$E_K = \frac{1}{2} \ L \ I^2 \ . \tag{5}$$

Comparing eqs.(4) and (5) reveals that the passage of the electron's mass induces a self-inductance too, given by

$$L_k = \frac{m\ell}{ne^2 A} \ , \tag{6}$$

which can be called inertial self-inductance. This quantity is however called the *kinetic self-inductance* too[4, 5]. Thus, the conductor appeared as if it were an inductor of length ℓ and cross-sectional area A. The inertial self-inductance in some cases can outnumber (exceed) the magnetic inductance. This occurs when A becomes exceedingly small. It is the case for nano-wires.

Interestingly, to connect the inertial self-inductance to the plasma frequency of the electron, ω_p, inside the conductor as

$$L_k = \frac{1}{\omega_p^2 C} \ , \qquad \omega_p = \sqrt{\frac{ne^2}{\varepsilon_0 m}} \ , \qquad C = \frac{\varepsilon_0 A}{\ell} \ , \tag{7}$$

where the conductor acted as if it were a parallel - plate capacitor. Interestingly, the plasma frequency is the resonance frequency of the equivalent $L_k - C$ circuit associated with the conductor. The impedance of the conductor is defined by $Z_k = \sqrt{\frac{L_k}{C}}$. Upon using eqs.(6) and (7) it transforms to

$$Z_k = \frac{\ell}{\epsilon_0 A \ \omega_p} \ , \qquad Z_k = \frac{1}{\omega_p \ C} \ . \tag{8}$$

Notice that in transmission lines the electromagnetic energy travels down the lines (wires) in a damped fashion. It seems that at the high magnetic field (current) the energy can no longer be contained inside the wire, and the wire explodes giving off its energy content. You know that transmission lines carry electromagnetic energy of low frequency only. When the frequency becomes very high, the wire acts as an antenna and radiates its energy away in electromagnetic waves in space.

Recall that the magnetic self-inductance of an inductor (coil) of length ℓ and cross-sectional area A and having N_t number of turns, is given by

$$L_B = \mu_0 N_t^2 \frac{A}{\ell} \ . \tag{9}$$

This shows that for nano-scales the self-inductance L_B becomes exceedingly small, but L_k becomes significant.

Let us now consider the Newton's second law of motion accounting for the motion of the electron's total mass that is given by

$$F = \frac{dp}{dt} = \frac{d}{dt} Mv \ , \tag{10}$$

which when one uses eqs.(1) and (3) yields

$$F = \frac{d}{dt}\left(\frac{m\ell \ I}{e}\right) \ . \tag{11}$$

This implies that the momentum the electron's mass carry is

$$p_e = \frac{m\ell}{e} \ I \ . \tag{12}$$

We better call p_e the electron's electric momentum. Recall that when an electron interacts with an electromagnetic field, its momentum is changed to become $p\ ' = p - qA_e$. One could reason our case by assuming that the charge's effect on the particle inertial motion is such that $-qA_e = p_e$. This states that the effect of the charge on the mass, as the particle moves, is as if being interacting with an electromagnetic field whose magnetic potential is $A_e = -\frac{m\ell}{e^2} \ I$, or $\vec{A}_e = -\frac{m\ell \ A}{e^2} \ \vec{J}$ or $\vec{J} = -\frac{ne^2}{Nm} \ \vec{A}$, where $n = N/(\ell A)$, and $J = I/A$ is the current density, and N is the total number of electrons. We call this potential the *inertial magnetic potential.*

It is pertinent to mention that London's current of superconductivity is given by $\vec{J} = -\frac{ne^2}{m} \ \vec{A}$[6] which is satisfied by the inertial magnetic potential above for $N = 1$. Thus, one may claim the supercurrent of London results from the electron's inertial (mass) motion, and not due to the charge motion as assumed. Intriguingly, mass and charge complement each other.

Recall that the magnetic flux, ϕ_B, is related to the magnetic self-inductance, L, by the relation

$$\phi_B = LI \quad , \tag{13}$$

so that the inertial magnetic flux, due to electrons' mass motion, will be

$$\Phi_k = L_k I = \frac{m\ell v}{e} \quad , \tag{14}$$

upon using eqs.(1) and (6). This is an interesting result showing that the flux arising from the electron's mass depends on the electron's mass besides its charge. Equation (14) defines the magnetic flux due to a single moving electron. We know that the electron's spin has a quantum origin, and that quantum mechanics deals with the mass aspect of the particle (de Broglie wave). Notice that the quantity $L = m\ell \; v$ has a unit of angular momentum. Can we associate this magnetic flux to the electron's spin?

The inertial magnetic field associated with the above flux can be obtained from the relation, $\phi_k = B_k A$. This yields,

$$B_k = \frac{m}{e}\frac{\ell}{A} \; v \quad , \qquad B_k = \frac{\ell\tau}{\sigma} \; \frac{J}{A} \quad , \qquad B_k = \frac{E}{v_0} \quad , \qquad v_0 = \frac{\ell\tau}{A} \quad , \qquad B_k = \frac{\tau}{A} \; V \quad , \qquad B_k = L_k J \quad , \tag{15}$$

using eq.(10), which is the magnetic field due to a single electron acting along the wire. Equation (15) expresses the Ohm's magnetic law,

$$J = \sigma_m B_k \quad , \qquad \sigma_m = \frac{1}{L_k} \quad , \tag{16}$$

where σ_m is the magnetic conductivity. Recall that the electric conductivity is the inverse of the electric resistivity, and eq.(16) has a similar behavior where the magnetic resistivity is but the kinetic inductance.

The above magnetic field acts along the wire axis (velocity). This is pertinent to the magnetic field in the waveguide (TM waves) where the magnetic field component along the wave direction is allowed to be present. From symmetric Maxwell's equations a longitudinal magnetic field results from a magnetic current (moving magnetic charge)[7]. The magnetic field under normal conditions is exceedingly small but grows larger for microscopic scales and high-velocity currents. Therefore, the magnetic monopole (magnetic charge) is a macroscopic phenomenon and does not appear under normal conditions. This reveals a longitudinal magnetic field is a high-energy phenomenon.

The magnetic force on the moving electrons can be deduced from $F = qvB_k$ which upon using eq.(15), yields

$$F_\ell = \left(\frac{m\ell}{A}\right) \; v^2 \; . \tag{17}$$

Interestingly, this self-force is a pure inertial force independent of the electron's charge. Thus, electron's acceleration is given by $a = \frac{\ell}{A} \; v^2$. It is a nonlinear resistive (drag) force. Because of this force, the wire experiences a tensile force along its length. It, therefore, appears that as if the electron is moving in a medium (viscous fluid). Thus, the motion of the electrons inside the conductor is analogous to the motion of a fluid in the pipeline. This force acts along the wire axis. It depends on the geometry of the conductor in which electrons move. The above tensile force on a smaller diameter wire is bigger than the thicker one. Notice eq.(17) is analogous to a force acting on a particle undergoing curved motion with a radius of curvature given by $r_c = A\ell^{-1}$. Does that mean the wire will bend by this amount?

Applying eq.(1) in eq.(17), upon using eq.(6), yields

$$F_\ell = \left(\frac{m\ell}{n^2e^2A^3}\right) \; I^2 = \frac{1}{2} \; \mu_k \; I^2 \; , \qquad F_\ell = \frac{\ell e^2\tau^2}{mA} \; E^2 \; , \qquad \mu_k = \frac{2}{nA^2} \; L_k \; , \tag{18}$$

where μ_k is the effective permeability of the medium (conductor), τ is the relaxation time of electrons, and E is the applied electric field. This force is obtained in the framework of the motion of massive photons which act like magnetic monopoles[8]. A longitudinal tensile force is found to have the same form, *viz*., $F = \frac{1}{2} \; \mu_0 I^2$. Whereas μ_0 is fixed, μ_k depends on the wire geometry. This force which is inversely proportional to the wire cross-sectional area tends to shatter the wire into pieces, which is not due to heat generated in the wire (it is sometimes referred to as Ampere's longitudinal force [9] and references therein). Notice that while the force in mechanics is directly proportional to the particle mass, the force in eq.(18) is inversely proportional to the electron mass.

Upon employing eq.(1) in the expression for B_k in eq.(15), the total magnetic fields becomes

$$B_k = \left(\frac{m\ell}{ne^2A^2}\right)I. \tag{19}$$

Intriguingly, the above magnetic field depends on the electron's mass that qualifies it to be called an *inertial magnetic field*. The magnetic flux associated with this magnetic field will be

$$\Phi_k = \frac{L}{e} \ , \tag{20}$$

if the angular momentum of the electron is quantized, *i.e.*, $L = s \ \ h$, where $s = 1,2,3,\cdots$ is an integer, then the inertial magnetic flux will be

$$\Phi_k = \frac{s \ \ h}{e} \ . \tag{21}$$

Therefore, eq.(20) yields

$$e \ \ \Phi_k = s \ \ h \ . \tag{22}$$

This is reminiscent of the Dirac's quantization rule for a monopole, *viz.,* $q_m q_e = sh$, where q_e and q_m are the electric and magnetic charges of the monopole [10]. Recall that the inertial motion of the electron is akin to quantum mechanics where the particle possesses a wave aspect. The charge motion gives rise to an electromagnetic wave. Equation (21) suggests that Φ_k has a flux quantum of h/e which is also found to be the case in electric (magnetic) systems.

3 Massive electrodynamics

The Maxwell electrodynamics in vacuum is associated with a Lagrangian of the form

$$L = \frac{1}{2}\varepsilon_0 E^2 - \frac{B^2}{2\mu_0} \ . \tag{23}$$

and the massive electrodynamics is defined by

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{\zeta}(\partial_\mu A^\mu)^2 + \frac{1}{2} \ \ m^2 A^\mu A_\mu \ , \tag{24}$$

where ζ is some parameter[11]. The second term in the above Lagrangian is added to help obtain a full canonical quantization [12]. However, we will show here that this term arises from considering massive electrodynamics without adding a term $m^2 A_\mu A^\mu$ that breaks the gauge symmetry. To this end, we treat the gauge field, A_μ, as the wavefunction of the photon and employ the momentum eigen-value equation of the photon. We employ quaternions to come to this electrodynamics. Quaternions are generalization of complex numbers. The quaternionic

Dirac momentum-eigen value equation, with β and $\vec{\gamma}$ as the Dirac matrices, is given by [8]

$$\tilde{\gamma}\tilde{P}^*\tilde{A} = mc\tilde{A} \ , \qquad \tilde{P}^* = (\frac{i}{c} \ E \ , -\vec{p}), \qquad \tilde{A} = (\frac{i}{c}\varphi \ , \vec{A}) \ , \qquad \tilde{\gamma} = (i\beta \ , \vec{\gamma}) \ , \qquad (25)$$

where m is the photon's mass, E and $\vec{p}$ are the energy and momentum of the photon. A quaternion $\tilde{A}$ is represented by $\tilde{A} = (a_0 \ , \vec{a})$, where a_0 is called the scalar part and $\vec{a}$ is called the vector part. The product of two quaternions, $\tilde{A}$ and $\tilde{B}$, is given by

$$\tilde{A}\tilde{B} = (a_0 b_0 - \vec{a}\cdot\vec{b} \ , a_0\vec{b} + \vec{a}b_0 + \vec{a}\times\vec{b}) \ . \qquad (26)$$

Applying the above quaternion product rule in eq.(25) yields the scalar part as

$$i\beta(\vec{p}\cdot\vec{A} - \frac{E}{c^2} \ \varphi) - \vec{\gamma}\cdot(\frac{i}{c}(E\vec{A} - \vec{p} \ \varphi) - \vec{p}\times\vec{A}) = im\varphi \ , \qquad (27)$$

and the vector part as

$$-\frac{\beta}{c}(E\vec{A} - \vec{p} \ \varphi) - i\beta\vec{p}\times\vec{A} + \vec{\gamma} \ (\vec{p}\cdot\vec{A} - \frac{E}{c^2} \ \varphi) + \frac{i}{c} \ \vec{\gamma}\times(E\vec{A} - \vec{p} \ \varphi) - \vec{\gamma}\times(\vec{p}\times\vec{A}) = mc\vec{A} \ . \qquad (28)$$

However, in quantum mechanics, $\vec{p} = -i\hbar\vec{\nabla}$ and $E = i\hbar\frac{\partial}{\partial t}$, so that eq.(27) yields

$$\vec{v}\cdot\vec{B} = -\frac{mc\beta}{\hbar}\varphi \ , \qquad \vec{v}\cdot\vec{E} = c^2\left(\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right), \qquad (29)$$

and eq.(28) yields

$$\vec{B} = \frac{\vec{v}}{c^2}\times\vec{E} - \frac{mc\beta}{\hbar}\vec{A} \ , \qquad \vec{E} = -\vec{v}\times\vec{B} + \vec{v}\left(\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right), \qquad (30)$$

where

$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial\vec{A}}{\partial t} \ , \qquad \vec{B} = \vec{\nabla}\times\vec{A} \ , \qquad \vec{v} = \beta c\vec{\gamma} \ . \qquad (31)$$

Interestingly, when $m = 0$, we restore the ordinary electromagnetic field in a frame moving with velocity $\vec{v}$. We see that taking the dot product of eq.(30) by $\vec{v}$ and compare the resulting equations with eq.(29) yields $v = c$ and $\vec{v}\cdot\vec{A} = \varphi$.

Equations (30) can be expressed as

$$\vec{B}' = \vec{B} - \frac{mc\beta}{\hbar}\vec{A} \ , \qquad \vec{E}' = \vec{E} + \vec{v}\left(\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right). \qquad (30a)$$

The electric field in eq.(30) can be expressed as

$$\vec{E}' = \vec{E} + \left(\frac{\vec{v}}{c^2}\cdot\vec{E}\right)\vec{v} \ , \qquad (30b)$$

using eq.(29). Evidently, the mass of the photon influences the electromagnetic field. It is found that the Lorenz gauge condition for massive electromagnetic field becomes [13]

$$\lambda=\vec{\nabla}\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}=\frac{m}{\hbar}\varphi\ . \tag{32}$$

The above gauge condition reduces to the Lorenz gauge condition for $m=0$. Note that one can see the electric and magnetic fields of photons due to their mass are

$$\vec{B}_{ph}=-\frac{mc}{\hbar}\vec{A}\ ,\qquad \vec{E}_{ph.}=\frac{m\varphi}{\hbar}\ \vec{v}\ ,\qquad \beta=\pm1\ . \tag{30c}$$

Applying eq.(32) in eq.(30a) yields

$$\vec{B}'=\vec{B}-\frac{mc\beta}{\hbar}\vec{A}\ ,\qquad \vec{E}'=\vec{E}+\frac{m\varphi}{\hbar}\ \vec{v}\ . \tag{30d}$$

The above electromagnetic field yields massive electrodynamics. Now the electromagnetic field due to massive photons is given by eqs.(29) and (30).

Now the Lagrangian of the new electrodynamics connecting the new massive fields will be

$$L=\frac{1}{2}\varepsilon_0E'^2-\frac{B'^2}{2\mu_0}\ . \tag{33}$$

Applying eqs.(30b) in (33), using eqs.(29) and (32), yields

$$L=\frac{1}{2}\varepsilon_0E^2-\frac{B^2}{2\mu_0}+\frac{1}{2}\ m^2(c^2A^2-\varphi^2)+\frac{\lambda^2}{2\mu_0}\ . \tag{34}$$

The Lagrangians in eq.(24) and (34) are those ones for massive electrodynamics. The above new formulation was not considered in the standard massive electrodynamics.

4 Massive field Lorentz force

Let us consider the Lorentz force acting on a moving charge in a massive electromagnetic field. This is given by the quaternionic equation

$$\tilde{F}=q\tilde{V}(\tilde{\nabla}\tilde{A})\ ,\qquad \tilde{F}=\left(\frac{i}{c}P-\frac{P_m}{c}\ ,\vec{f}+i\vec{f}_m\right)\ ,\qquad \tilde{V}=(ic\ ,\vec{v})\ ,\qquad \tilde{\nabla}=\left(\frac{i}{c}\frac{\partial}{\partial t}\ ,\vec{\nabla}\right)\ , \tag{35}$$

which upon using eq.(25), yields

$$P=q\vec{v}\cdot\vec{E}-qc^2\lambda\ ,\qquad P_m=qc\vec{v}\cdot\vec{B}\ , \tag{36}$$

$$\vec{f}=q(\vec{E}+\vec{v}\times\vec{B}-\lambda\ \vec{v})\ ,\qquad \vec{f}_m=qc(\vec{B}-\frac{\vec{v}}{c^2}\times\vec{E})\ . \tag{37}$$

Notice that the force $\vec{f}_m$ is a magnetic force acting on a magnetic charge, $q_m=cq$. Notice that the magnetic power is associated with a magnetic charge that would necessitate the existence of magnetic monopoles in Maxwell's equations. The disintegration of magnetic charges could also lead to a huge magnetic force along the wire axis. The modified Lorentz force, eq.(37), involves an additional force proportional to the particle velocity. This term would presume a presence of a fluid permeating the whole space. This force exists even no electromagnetic field is present. We restore the standard electrodynamics when the field mass is set to zero.

5 Concluding remarks

We have derived in this work the inertial electromagnetic quantities arising from the electron and photon motions. In this scenario, a longitudinal tensile force is generated whenever a large current is passed in a wire. This inertial electrodynamics could also emerge from considering a massive electromagnetic field that extends Maxwell's equations allowing magnetic monopole to exist. Interestingly, the Lagrangian associated with massive photons is shown to yield that one previously obtained. The latter massive electrodynamics is due to Proca and de Broglie.

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