Journal of Qassim University for Science
2023, *Vol 2* (Issue 2), pp. 147-152
Received: 1 June 2023
Revised: 13 June 2023
Accepted: 16 June 2023

JQUS

On Minimizing Machines Rental Cost Using Heptagonal Fuzzy Processing Time

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Abstract: This paper is concerned with minimizing the total rental cost of special structure three phases flow shop scheduling problem. We propose a new approach to solve such problem with heptagonal fuzzy processing time. The heptagonal fuzzy processing time is processed without transforming it to a deterministic number. For demonstrating the efficiency of the proposed approach, a numerical example is illustrated.

Keywords: Flow Shop Scheduling; Heptagonal Fuzzy Numbers; Fuzzy Processing Time; optimization problem; decision making.

1. INTRODUCTION

The flow shop scheduling problem (FSSP) is defined as, how to arrange n-jobs on m-machines. Jobs are processed sequentially through all machines, with each machine handling just one job at a time. Many researchers are still research in this area [1–6]. Ueno et al. investigated the steel industry's multi-stage flow-shop issue in [7]. Yuan et al. [8] examine the development and modeling of an algorithm for the two-stage FSSP with a specific blocking restriction.

Zadeh introduced the idea of fuzzy set theory in 1965 [9]. Fuzzy sets are more flexible in quantifying and finding a solution by examining vague concepts. In the past six decades, several varieties of fuzzy numbers have emerged, for instance, triangular [10], trapezoidal [11], pentagonal [12], etc. Heptagonal fuzzy number (HFN) was introduced for the first time by K. Rathi and Belmopan [13]. We often make use of fuzzy numbers to describe the processing times, as there is a shortage in knowledge about them. Many researchers focus their attention on studying Fuzzy flow shop scheduling problem. Johnson's [14] devised an algorithm for reducing the completion time of all tasks in two and three machines. Sathish and Ganesan [10] explored a strategy to reduce the rental cost of three machines with fuzzy processing time under a specific policy. EL-Morsy et al. [15] investigated a scheduling problem involving Pythagorean fuzzy environment. Khalifa et al. [16] studied a constrained FSSP using a multi-stage fuzzy binding technique with fuzzy due dates. In [17], Alharbi and Khalifa provided an approach for solving a FSSP involving pentagonal processing time. The main goal of this paper, is to propose an approach for minimizing the total rental cost of three stages FSSP having heptagonal fuzzy processing time.

The structure of this paper is as follows. In Section 2, the definition and the arithmetic operations of heptagonal fuzzy number are given. Section 3, illustrates the proposed algorithm. Section 4, formulates the problem. Section 5, presents a numerical application to illustrate the efficiency of the proposed approach. Section 6, is a comparative study to compere the obtained results. Section 7, gives some conclusion remarks.

2. HEPTAGONAL FUZZY NUMBER (HFN)

This section should provide sufficient details of materials and methods to allow the readers to replicate the published results and build on further research. As the publication of your manuscript includes that all materials, data, codes, and protocols associated with the publication should be available to the readers. Any limitations on the provision of this information should be disclosed at the submission stage. See data availability section at the end of this document. Please describe the new methods, frameworks, and approaches developed in your research in detail, while briefly outline the well-documented methods with appropriate references.

2.1 Definition, Mathematical and Geometric Representation of HFN

Definition 1: (Heptagonal Fuzzy Number)

A fuzzy number $\tilde{A}_h$ is a pentagonal fuzzy number shown in figure1 denoted by $\tilde{A}_h = (h_1, h_2, h_3, h_4, h_5, h_6, h_7; k, w)$ where $h_1, h_2, h_3, h_4, h_5, h_6$ and h_7 are real numbers, $0 \leq k \leq w \leq 1$ and its membership function $\mu_{\tilde{A}_h}(y)$ is given below[13].

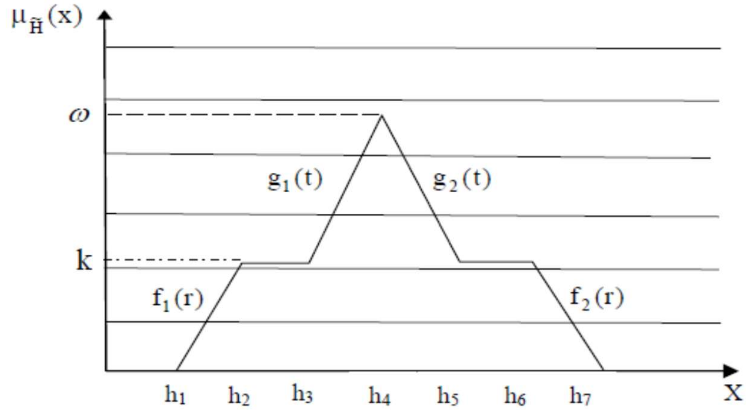

Figure 1: Geometric representation of HFN [13]

$$\mu_{\tilde{A}_h}(y, w, v) = \begin{cases} 0, & y < h_1 \\ k\dfrac{y - h_1}{h_2 - h_1}, & h_1 \leq y \leq h_2 \\ k, & h_2 \leq y \leq h_3 \\ k + (w - k)\dfrac{y - h_3}{h_4 - h_3}, & h_3 \leq y \leq h_4 \\ k + (w - k)\dfrac{h_5 - y}{h_5 - h_4}, & h_4 \leq y \leq h_5 \\ k, & h_5 \leq y \leq h_6 \\ k\dfrac{y - h_7}{h_6 - h_7}, & h_6 \leq y \leq h_7 \\ 0, & y \geq h_7 \end{cases}$$

Remarks

1. When $w = 1$, the above HFN is converted to the normal HFN.
2. When $k = 0$, HFN reduces to triagonal fuzzy number i.e. $(h_1, h_2, h_3, h_4, h_5, h_6, h_7; k, w) \cong (h_3, h_4, h_5; w)$.
3. When $k = 1$, HFN reduces to trapezoidal fuzzy number i.e. $(h_1, h_2, h_3, h_4, h_5, h_6, h_7; k, w) \cong (h_1, h_2, h_6, h_7; w)$.

1.1 Arithmetic Operations on HFN

Let $\tilde{A}_h = (h_1, h_2, h_3, h_4, h_5, h_6, h_7)$ and $\tilde{B}_h = (p_1, p_2, p_3, p_4, p_5, p_6, p_7)$ be two HFNs, every HFN associated with two weights k, w. To avoid confusion, we use the notations k_1, w_1 to represent the weights of $\tilde{A}_h$, and k_2, w_2 to represent the weights of $\tilde{B}_h$. The arithmetic operations o $\tilde{A}_h$ and $\tilde{B}_h$ can be defined as follow [13].

Addition: $\tilde{A}_h + \tilde{B}_h = (h_1 + p_1, h_2 + p_2, h_3 + p_3, h_4 + p_4, h_5 + p_5, h_6 + p_6, h_7 + p_7)$ with $k_3 = \min(k_1, k_2)$ and $w_3 = \min(w_1, w_2)$.

Subtraction: $\tilde{A}_h - \tilde{B}_h = (h_1 - p_7, h_2 - p_6, h_3 - p_5, h_4 - p_4, h_5 - p_3, h_6 - p_2, h_7 - p_1)$ with $k_3 = \min(k_1, k_2)$ and $w_3 = \min(w_1, w_2)$.

Scalar Multiplication: let c be a real number. If $c \geq 0$, $c\,\tilde{A}_h = (c\,h_1, c\,h_2, c\,h_3, c\,h_4, c\,h_5, c\,h_6, c\,h_7)$, if $c \leq 0$, $c\,\tilde{A}_h = (c\,h_7, c\,h_6, c\,h_5, c\,h_4, c\,h_3, c\,h_2, c\,h_1)$.

Multiplication: $\tilde{A}_h\tilde{B}_h = (h_1p_1, h_2p_2, h_3p_3, h_4p_4, h_5p_5, h_6p_6, h_7p_7)$ with $w_3 = \min(w_1, w_2)$ and $k_3 = \min(k_1, k_2)$

Inverse: $\tilde{A}_h^{-1} = (\frac{1}{h_7}, \frac{1}{h_6}, \frac{1}{h_5}, \frac{1}{h_4}, \frac{1}{h_3}, \frac{1}{h_2}, \frac{1}{h_1})$, $h_i \neq 0, 1 \leq i \leq 7$. If one or more of $a_i = 0, 1 \leq i \leq 7$, then we can't find the inverse of the HFN.

Division: the division of two HFN is defined as, $\frac{\tilde{A}_h}{\tilde{B}_h} = \tilde{A}_h\tilde{B}_h^{-1} = \left(\frac{h_1}{p_7}, \frac{h_2}{p_6}, \frac{h_3}{p_5}, \frac{h_4}{p_4}, \frac{h_5}{p_3}, \frac{h_6}{p_2}, \frac{h_7}{p_1}\right)$, $k_3 = \min(k_1, k_2)$ and $w_3 = \min(w_1, w_2)$.

Exponent: The exponent of a HPN, $\tilde{A}_h$ is defined as, $\tilde{A}_h^{\,n} = (h_1^{\,n}, h_2^{\,n}, h_3^{\,n}, h_4^{\,n}, h_5^{\,n}, h_6^{\,n}, h_7^{\,n})$ with n being a real number.

3. Assumptions, Rental Policy and Notations

In this section three stages special structured fuzzy flow-shop scheduling problem is studied with the following assumptions.

1. Pre-emption is not allowed to any job.
2. A unique job can be served at a time.
3. In the start of the schedule time, all jobs are available.
4. Neglect the setting up times of all machines.
5. All jobs are processed throughout the deterministic phase.
6. Due dates are HFNs.
7. The machines may be idle.
8. The production period is independent on the schedule.
9. For every task, m operations are required.
10. Once a job is begun, it must be finished.

The first job must be finished in the first machine to be processed on the second machine and so on.

4. Problem Formulation

Let the i^{th} job, $i = 1{:}n,$ is to be served on the j^{th} machine, $j = 1{:}m$ with a given rental cost $\mathcal{P}$. Define $\widetilde{\mathcal{B}}_{ij}$ as the HFN processing time of i^{th} job on j^{th} machine. The goal is to determine the optimal sequence $\{\mathcal{S}_k\}$ of jobs for minimizing the rental cost. We can formulate this problem as follows:

Minimize $\widetilde{\Re}(\mathcal{S}_k) = \sum_{i=1}^{n}[\widetilde{\mathcal{B}}_{i1}\,\widetilde{\mathcal{H}}_1 + \widetilde{\mathcal{B}}_{i2}\,(\mathcal{S}_k)\widetilde{\mathcal{H}}_2 + \widetilde{\mathcal{B}}_{i3}\,(\mathcal{S}_k)\widetilde{\mathcal{H}}_3\,]$, $\widetilde{\mathcal{H}}_k$ is the utilization time of the k^{th} machine, $k = 1{:}3$,

subject to: the given rental policy $\mathcal{P}$.

Assume $\widetilde{\mathcal{B}}_{i1}, \widetilde{\mathcal{B}}_{i2}, \widetilde{\mathcal{B}}_{i3}, \dots, \mathcal{B}_{im}$ be the HFN processing times of machines $\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3, \dots, \mathcal{N}_m$ respectively, we propose the following algorithm:

***Step 1*:** The three machines problem can be transformed to two machines problem, if we have one of the two listed conditions:

- $\min_i \widetilde{\mathcal{B}}_{i1} \geq \max_i \widetilde{\mathcal{B}}_{ij}\,, j = 2{:}m-1,$
- $\min_i \widetilde{\mathcal{B}}_{im} \geq \max_i \mathcal{B}_{ij}\,, j = 2{:}m-1.$

***Step 2*:** Consider two machines $\mathcal{X}$ and $\mathcal{Y}$ such that

$$\widetilde{\mathcal{X}}_i = \sum_{j=1}^{m-1} \widetilde{\mathcal{B}}_{ij}\,, i = 1{:}n.$$

$$\widetilde{\mathcal{Y}}_i = \sum_{j=2}^{m} \widetilde{\mathcal{B}}_{ij}\,, i = 1{:}n.$$

where, $\widetilde{\mathcal{X}}_i, \widetilde{\mathcal{Y}}_i$ are HFN processing time of job i on machines $\mathcal{X}$ and $\mathcal{Y}$ respectively.

Step 3: Evaluate the sequence of jobs $\{\mathcal{S}_k\}$ on machines $\mathcal{X}$ and $\mathcal{Y}$ using ranking method.

5. Numerical Application

In a certain factory, we have 3 machines and 5 jobs to do on these machines. Consider a FSSP with heptagonal fuzzy processing time shown in table 1. The rental cost of machines $\mathcal{N}_1, \mathcal{N}_2$ and $\mathcal{N}_3$ per unit time are 4 units, 2 units and 3 units respectively, under the rental policy $\mathcal{P}$ [10]. The main goal is to obtain an optimal schedule of the given jobs.

Table1: HFN processing time

Jobs	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$
1	(7, 7.3, 7.6, 8,8.3,8.6,9)	(6, 6.3, 6.7, 7, 7.3, 7.6, 8)	(3,3.3,3.6,4,4.3,4.6,5)
2	(12, 12.3, 12.6, 13, 13.3, 13.6, 14)	(5, 5.1, 5.8, 6, 6.1, 6.8, 7)	(4, 4.2, 4.8, 5, 5.2, 5.8, 6)
3	(8, 8.5, 9, 10, 11, 11.5, 12)	(4, 4.2, 4.6, 5, 5.2, 5.6, 6)	(6, 6.2, 6.87, 7.2, 7.8, 8)
4	(10, 10.3, 10.6, 11, 11.3, 11.6, 12)	(5,5.3,5.7,6,6.3,6.7,7)	(11,11.3,1.4,12,12.3,12.4,13)
5	(9, 9.2, 9.7, 10, 10.2, 10.7, 11)	(5, 5.2, 5.8, 6, 6.2, 6.8, 7)	(8, 8.2, 8.9, 9, 9.2, 9.9, 10)

For step1 of the algorithm,

$$\min_i \widetilde{\mathcal{B}}_{i1} = (7, 7.3, 7.6, 8, 8.3, 8.6, 9),$$
$$\max_i \widetilde{\mathcal{B}}_{i2} = (6, 6.3, 6.7, 7, 7.3, 7.6, 8),$$
$$\min_i \widetilde{\mathcal{B}}_{i3} = (3, 3.3, 3.6, 4, 4.3, 4.6, 5).$$

As $\min_i \widetilde{\mathcal{B}}_{i1} > \max_i \widetilde{\mathcal{B}}_{i2}$, then we can convert the problem to two machines. Let $\mathcal{X}$ and $\mathcal{Y}$ be two machines such that

$$\tilde{\mathcal{X}}_i = \sum_{j=1}^{2} \tilde{\mathcal{B}}_{ij}, \qquad \tilde{\mathcal{Y}}_i = \sum_{j=2}^{3} \tilde{\mathcal{B}}_{ij}$$

Table 2 : HFN processing times of machines $\mathcal{X}$ and $\mathcal{Y}$

Job	$\mathcal{X}$	$\mathcal{Y}$
1	(13, 13.6, 14.3, 15, 15.6, 16.2, 17)	(9, 9.6, 10.3, 11, 11.6, 12.2, 13)
2	(17, 17.4, 18.4, 19, 19.4, 20.4, 21)	(9, 9.3, 10.6, 11, 11.4, 12.5, 13)
3	(12, 12.7, 13.6, 15, 16.2, 17.1, 18)	(10, 10.4, 11.4, 12, 12.4, 13.4, 14)
4	(15, 15.6, 16.3, 17, 17.6, 18.3, 19)	(16, 16.6, 17.1, 18, 18.6, 19.1, 20)
5	(14, 14.4, 15.5, 16, 16.4, 17.5, 18)	(13, 13.4, 14.7, 15, 15.4, 16.7, 17)

the optimal sequence obtained by subinterval average method [18], is, $4 \rightarrow 5 \rightarrow 2 \rightarrow 3 \rightarrow 1$.

Table 3: Time in and time out of $\mathcal{N}_1$

Job	Time in	Time out
4	(0, 0, 0, 0, 0, 0, 0)	(10, 10.3, 10.6, 11, 11.3, 11.6, 12)
5	(10, 10.3, 10.6, 11, 11.3, 11.6, 12)	(19, 19.5, 20.3, 21, 21.5, 22.3, 23)
2	(19, 19.5, 20.3, 21, 21.5, 22.3, 23)	(31, 31.8, 32.9, 34, 34.8, 35.9, 37)
3	(31, 31.8, 32.9, 34, 34.8, 35.9, 37)	(39, 40.3, 41.9, 44, 45.8, 47.4, 49)
1	(39, 40.3, 41.9, 44, 45.8, 47.4, 49)	(46, 47.6, 49.5, 52, 54.1, 56, 58)

Table 4: Time in and time out of $\mathcal{N}_2$

Job	Time in	Time out
4	(10, 10.3, 10.6, 11, 11.3, 11.6, 12)	(15, 15.6, 16.3, 17, 17.9, 18.3, 19)
5	(19, 19.5, 20.3, 21, 21.5, 22.3, 23)	(24, 24.7, 26.1, 27, 27.7, 29.1, 30)
2	(31, 31.8, 32.9, 34, 34.8, 35.9, 37)	(36, 36.9, 38.7, 40, 40.9, 42.7, 44)
3	(39, 40.3, 41.9, 44, 45.8, 47.4, 49)	(43, 44.5, 46.5, 49, 51, 53, 55)
1	(46, 47.6, 49.5, 52, 54.1, 56, 58)	(52, 53.9, 56.2, 59, 61.4, 63.6, 66)

Table 5: Time in and time out of $\mathcal{N}_3$

Job	Time in	Time out
4	(15, 15.6, 16.3, 17, 17.9, 18.3, 19)	(26, 26.9, 27.7, 29, 30.2, 30.7, 32)
5	(26, 26.9, 27.7, 29, 30.2, 30.7, 32)	(34, 35.1, 36.6, 38, 39.4, 40.6, 42)
2	(36, 36.9, 38.7, 40, 40.9, 42.7, 44)	(40, 41.1, 43.5, 45, 46.2, 48.4, 50)
3	(43, 44.5, 46.5, 49, 51, 53, 55)	(49, 50.7, 53.3, 56, 58.2, 60.8, 63)
1	(52, 53.9, 56.2, 59, 61.4, 63.6, 66)	(55, 57.2, 59.8, 63, 65.7, 68.2, 71)

From tables 3-5, we can notice that:

The time necessary to complete all tasks in the chosen order, $\mathcal{HT}(\mathcal{S}_i)$ =(55, 57.2, 59.8, 63, 65.7, 68.2, 71) Idle time of $\mathcal{N}_1$ is,

$\tilde{\mathcal{I}}_1 = (55, 57.2, 59.8, 63, 65.7, 68.2, 71) - (46, 47.6, 49.5, 52, 54.1, 56, 58)$
$= (-3, 1.2, 5.7, 11, 16.2, 20.6, 25)$

Idle time of $\mathcal{N}_2$ is,

$\tilde{\mathcal{I}}_2 = (0, 1.2, 2.4, 4, 5.2, 6.7, 8) + (1, 2.7, 5.2, 7, 8.7, 12, 13) + (-5, -2.4, 1, 4, 7.1, 10.5, 13) + (-9, -5.4, -1.5, 3, 7.6, 11.5, 15) = (-13, -3.9, 7.1, 18, 28.5, 39.9, 49)$

Idle time of $\mathcal{N}_3$ is,

$\tilde{\mathcal{I}}_3 = (-6, -3.7, -0.7, 2, 4.3, 7.6, 10) + (-7, -3.9, 0.3, 4, 7.5, 11.9, 15) + (-11, -6.9, -2, 3, 8.1, 12.9, 17) = (-24, -14.5, -2.4, 9, 19.9, 32.4, 42)$.

Now we are going to evaluate the utilization time of the three machines $\tilde{\mathcal{V}}_i, i = 1,2,3$

$\tilde{\mathcal{V}}_1 = (46, 47.6, 49.5, 52, 54.1, 56, 58)$ hrs.

$\tilde{\mathcal{V}}_2 = (52, 53.9, 56.2, 59, 61.4, 63.6, 66) - (-13, -3.9, 7.1, 18, 28.5, 39.9, 49) = (3, 14, 27.6, 41, 54.3, 67.5, 79)$hrs.

$\tilde{\mathcal{V}}_3 = (55, 57.2, 59.8, 63, 65.7, 68.2, 71) - (-24, -14.5, -2.4, 9, 19.9, 32.4, 42) = (13, 24.8, 39.9, 54, 68.1, 82.7, 95)$ hrs.

The final step is computing the renal cost of machines,

$\tilde{\mathcal{H}}_1 = 4 * (46, 47.6, 49.5, 52, 54.1, 56, 58) = (184, 190.4, 198, 208, 216.4, 224, 232)$ units.

$\mathcal{H}_2 = 2 * (3, 14, 27.6, 41, 54.3, 67.5, 79) = (6, 28, 55.2, 82, 108.6, 135, 158)$ units.

$\tilde{\mathcal{H}}_3 = 3 * (13, 24.8, 39.9, 54, 68.1, 82.7, 95) = (39, 74.4, 119.7, 162, 204.3, 248.1, 285)$ units.

Total rental cost, $\tilde{\mathfrak{R}}(\mathcal{S}_k)$=$\sum_{j=1}^{3} \tilde{\mathcal{H}}_j$= (229, 292.8, 372.9, 452, 529.3, 607.1, 675) units.

The above results can be summarized in the table 6:

Table 6: Idle time, Utilization time and Rental cost

Item	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$
Idle time	$(-3, 1.2, 5.7, 11, 16.2, 20.6, 25)$	$(-13, -3.9, 7.1, 18, 28.5, 39.9, 49)$	$(-24, -14.5, -2.4, 9, 19.9, 32.4, 42)$
Utilization time	$(46, 47.6, 49.5, 52, 54.1, 56, 58)$	$(3, 14, 27.6, 41, 54.3, 67.5, 79)$	$(13, 24.8, 39.9, 54, 68.1, 82.7, 95)$
Rental cost	(8, 8.5, 9, 10, 11, 11.5, 12)	(4, 4.2, 4.6, 5, 5.2, 5.6, 6)	(6, 6.2, 6.87, 7.2, 7.8, 8)

6. COMPARATIVE STUDY

This section aims to examine the accuracy of the results obtained by the proposed algorithm. We compare our obtained results the total processing time, the total rental cost of machines and Idle time of machines with those results obtained by Sathish and Ganesan in [10]. The HFN is a generalized form of the triangular fuzzy number. Tables 7-9, indicates this comparison.

Table 7: Processing time

Type of fuzzy number	Proposed approach	[10]
Heptagonal	(55, 57.2, 59.8, 63, 65.7, 68.2, 71)	(61, 61.5, 62, 63, 64, 64.5, 65)
Triangular	(59.8, 63, 65.7)	(61, 63, 65)
LR Triangular	(62.75, 2.95, 2.95)	(63, 2, 2)

Table 8: Total Renal cost

Type of fuzzy number	Proposed approach	[10]
Heptagonal	(229, 292.8, 372.9, 452, 529.3, 607.1, 675)	(444, 446, 448, 452, 456, 458, 460)
Triangular	(372.9, 452, 529.3)	(444, 452, 460)
LR Triangular	(451.1, 78.2, 78.2)	(452, 8, 8)

Table 9: Idle time of machines

Type of fuzzy number	$\mathcal{J}_j$	Proposed approach	[10]
Heptagonal	$\mathcal{J}_1$	$(-3, 1.2, 5.7, 11, 16.2, 20.6, 25)$	(9, 9.5, 10, 11, 11.5, 12, 13)
	$\mathcal{J}_2$	$(-13, -3.9, 7.1, 18, 28.5, 39.9, 49)$	(16, 16.5, 17, 18, 18.5, 19, 20)
	$\mathcal{J}_3$	$(-24, -14.5, -2.4, 9, 19.9, 32.4, 42)$	(7, 7.5, 8, 9, 9.5, 10, 11)
Triangular	$\mathcal{J}_1$	$(5.7, 11, 16.2)$	(9, 11, 13)
	$\mathcal{J}_2$	$(7.1, 18, 28.5)$,	(16, 18, 20)
	$\mathcal{J}_3$	$(-2.4, 9, 19.9)$	(7, 9, 11)
LR Triangular	$\mathcal{J}_1$	(10.95, 5.25, 5.25)	(11, 2, 2)
	$\mathcal{J}_2$	(17.8, 10.7, 10.7)	(18, 2, 2)
	$\mathcal{J}_3$	(8.75, 11.15, 11.15)	(9, 2, 2)

The above tables, shows that: When membership $\mu_A(x) = 1$, the value of x in the heptagonal and the triangular fuzzy numbers is equal**.**

7. CONCLUSION

An approach for solving the special structured three stages FSSP with HFN processing time is introduced. The total Rental cost of machines is evaluated. The obtained a differ sequencing of operations than the obtained in [14]. The subinterval's ranking method is used. In spite of the different sequencing order of jobs, similar results as those delivered in [10] are obtained. The values of left and right fuzziness index, are larger than the values obtained in [10], this for all computed values, i. e., the approach is more flexible as it increases the selecting decision region and enables the decision maker to select the suitable values according to his objective.

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Arabic Abstract

حول تدنية تكلفة استئجار الآلات بإستخدام وقت معالجة ضبابي سباعي الأضلاع

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تختص هذه الورقة بتقليل إجمالي تكلفة الإيجار للهيكل الخاص المكون من ثلاث مراحل لمشكلة جدولة ورشة العمل. نقترح نهجًا جديدًا لحل هذه المشكلة بوقت معالجة ضبابي سباعي الأضلاع. تتم معالجة وقت المعالجة الضبابي السباعي الشكل بدون تحويله إلى رقم حتمي. لتوضيح كفاءة النهج المقترح ، تم حل مثال رقمي.

الكلمات المفتاحية: جدولة التدفق, الأرقام السباعية الضبابية, وقت المعالجة الضبابي, مشاكل الأمثلية, اتخاذ القرار