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A moving load's influence on nanobeams resting on a two-parameter Pasternak foundation using the DPL model.

Manar AL mousa[1] and Doaa Atta[2,3]

[1] Department of Mathematics, College of Science and Arts in Unaizah, Qassim University, Saudi Arabia, Email: m.MOUSAA@qu.edu.sa

[2] Department of Mathematics, College of Science, Qassim University, P.O. Box 6644, Buraydah 51482, Saudi Arabia, d.mohamed@qu.edu.sa

[3] Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

Correspondence author: Doaa Atta[2,3]

Abstract

Thermoelastic vibrations of a nonlocal nanobeam lying on a two-parameter basis and subjected to a transverse moving load are introduced in the current study. For the linear Winkler-Pasternak foundation type, governing equations are established on the basis of the generalized dual-phase-lag heat conduction and nonlocal beams theories. Ramp-type changing heat is applied to the nanobeam. The Laplace transform approach is used to define and solve the problem's linked equations. A graphic is used to illustrate and discuss how the nonlocal parameter and several foundation parameters affect the field variables. The results are in line with previous analytical and numerical findings.

Key Words Foundation; Nanobeams; nonlocal thermoelasticity; moving load.

1. Introduction

The production of sophisticated materials at the nanoscale, which opens up a new class of structures with novel features and better performance devices, is the focus of nanotechnology. Due to their many potential uses as nanowires, nanoprobes, atomic power supplies, nano-actuators, and nano-sensors, among these nanostructures, nanobeams get more attention. Additionally, nanoscale effects have a key role in how well nanostructures with tiny diameters and spacing between molecules function mechanically. Many academics were motivated by this to develop a novel model to forecast the way that these nanostructures behave mechanically. There have been more studies on nonlocal theoretical models recently, including several forms of nonlocal elasticity techniques that include deeply researched hardening and softening models. Due to its universality and simplicity, Micro/nano-scale mechanical systems are simulated using the theory of nonlocal elasticity (NET). Eringen was the first to present this notion [1-3]. The strain field at each point along a continuum object is, in accordance with the NET theory is a function of the stress field at any particular location along the body. The nonlocal elasticity theory (NET) has been widely employed in research to account for Nanobeams and nanostructures are affected by vibration at the nanoscale. They can be found in references [4–14] in some cases. The design of airplane structures often includes nanobeams sitting on elastic foundations, which are widely used in structural analysis. Many scientists were motivated by this to investigate how constructions performed on various types of elastic foundations. The Winkler-type elastic foundation is thought to consist of a collection of vertical, linear elastic springs that are precisely spaced apart from one another. Transverse shear deformation and Winkler-type elastic springs make up the two parameters of the Pasternak model. Several authors [15-21] have investigated the effect of Winkler and Pasternak elastic foundations on the bending and vibration of micro-nanomaterials. The moving loads, on the other hand, have a big impact on how the engineered structure behaves dynamically. As moving forces (loads) in the form of automobile traffic exert pressure on the framework structures used in transport engineering, such as bridges, this causes vibration in the structures. Deflections and strains brought on by a moving vehicle on a bridge are often larger than those brought on by the same vehicle loads applied statically. There is a wealth of literature on the subject of dynamic analysis of a structure subject to a moving load.

Throughout the present research, we use Eringen's nonlocal elasticity and Tzou's thermoelastic dual-phase-lag model [22–24], which modifies the Fourier law, to examine the thermoelastic vibration of a nanobeam exposed to transverse moving load and resting on Winkler–Pasternak foundation. The nanobeam is thermally loaded by ramp-type shifting heat. A nanobeam exposed to ramp-type shifting heat is studied for its thermoelastic vibration in terms of temperature, deflection, displacement, and bending moment. Several comparisons were represented graphically to assess the effects of the nonlocal parameter and Winkler-Pasternak foundation parameters on all the field variables.

2. Materials and Methods:

2. The nonlocal thermoelasticity with phase lags

The differential nonlocal constitutive equations for a homogeneous thermoelastic material are given by Eringen's nonlocal elasticity theory [1-3]

$$(1-\xi\nabla^2)\sigma_{ij} = \tau_{ij}, \qquad (1)$$

where τ_{ij} and σ_{ij} represent both of the local and nonlocal stress tensors, respectively.

According to Tzou model [22-24], the generalized heat equation with a two-phase delay is as follows:

$$K\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\nabla^2\theta = \left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left[\rho C_E\frac{\partial\theta}{\partial t}+\gamma T_0\frac{\partial}{\partial t}\left(\mathrm{div}(\boldsymbol{u})\right)-Q\right]. \qquad (2)$$

The constitutive equations:

$$\tau_{ij} = 2\mu e_{ij} + \lambda e_{ij} - \gamma\theta\delta_{ij}\ . \qquad (3)$$

And, the equation of motion:

$$\sigma_{ji,j} + F_i = \rho\ddot{u}_i. \qquad (4)$$

By substituting ξ for zero in equation (1), we can readily construct the constitutive equation of classical local thermoelasticity, In this case, we ignored the intrinsic feature of length, which means that the particles of the medium are scattered in a continuous distribution.

3. The Problem's Formulation

Suppose a thermoelastic nanobeam such that its dimensions are $\mathbf{0} \le \mathbf{x} \le \mathbf{L}$, $\mathbf{0} \le \mathbf{y} \le \mathbf{b}$ and $\mathbf{0} \le \mathbf{z} \le \mathbf{h}$ where $\mathbf{L}$, $\mathbf{b}$ and $\mathbf{h}$ represent the nanobeam's length, width and thickness. As shown in Fig. 1, the initial temperature of nanobeam is $\mathbf{T_0}$ and it rests on a linear Winkler-Pasternak foundation, $\mathbf{K_w}$ and $\mathbf{K_s}$. In addition, we assume that the axial direction of the beam in this problem is $\mathbf{x}$ axis.

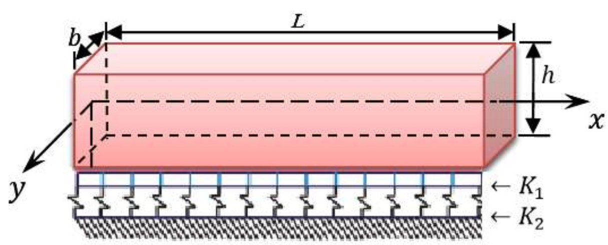

Fig. 1: Nanobeam schematic on the Pasternak foundation.

The displacement field at any arbitrary point of the beam can be taken as below:

$$u = -z\frac{\partial w}{\partial x}, v = 0,\ w(x, y, z, t) = w(x, t). \quad (5)$$

Relying on the previous assumption of displacement, we can derive the differential constitutive Eq. (3) for a one-dimensional problem by substituting these displacements into each of the Eqs. (1) and (5) to take the following form [8, 25]:

$$\sigma_{xx} - \xi\frac{\partial^2 \sigma_x}{\partial x^2} = -E\left[z\frac{\partial^2 w}{\partial x^2} + \alpha_T\theta\right], \quad (6)$$

where σ_{xx} refers to the nonlocal axial stress, and $\alpha_T = \alpha_t/(1 - 2\nu)$.

The equilibrium equation for microbeam transverse vibration is expressed as

$$\frac{\partial^2 M}{\partial x^2} - \rho A\frac{\partial^2 w}{\partial t^2} = 0, \quad (7)$$

where $A = bh$, refers to the cross-section area.

Winkler's elastic foundation model is the most introductory, as it assumes that at any arbitrary point, the vertical displacement is proportional to the contact pressure[26]. The normal stress per unit area R_f (the reaction of the foundation) and vertical displacement w at any point along the lower boundary of the nanobeam retain the following relationship as a result of the interaction between the nanobeam and the supporting foundation [27, 28]

$$R_f = K_w w(x, t) - K_s\frac{\partial^2 w(x,t)}{\partial x^2}, \quad (8)$$

where K_w and K_s stand for the Winkler's foundation constant, it is also referres to the modulus of substrate response, and the shear basis modulus respecitively.

In the case of putting $K_\mathrm{s} = 0$ in Eq. (8) we will get the relation in case of nanobeam on a Winkler foundation type; however, in the case of obtaining a nanobeam that has no foundation both of $K_\mathrm{w} = K_\mathrm{s} = 0$ is substituted

The transverse vibrational equation of motion for nanobeams is expressed as

$$\frac{\partial^2 M}{\partial x^2} - R_f = \rho A\frac{\partial^2 w}{\partial t^2} - q(x). \quad (9)$$

The flexure moment is determined using Eq. (6) and is given by

$$M(x, t) - \xi\frac{\partial^2 M}{\partial x^2} = -IE\left[\frac{\partial^2 w}{\partial x^2} + \alpha_T M_T\right], \quad (10)$$

where

$$M_T = \frac{12}{h^3}\int_{-h/2}^{h/2}\theta(x, z, t)z dz. \quad (11)$$

Additionally, it is clearly demonstrated that the nonlocal nanobeams' flexure moment is generate d by

$$M(x,t) = \xi A\rho \frac{\partial^2 w}{\partial t^2} + \xi K_w w(x,t) - (IE + \xi K_s)\frac{\partial^2 w(x,t)}{\partial x^2} - \alpha_{tt} M_T - \xi q(x), \tag{12}$$

where $\alpha_{tt} = IE\alpha_t$.

One may obtain the motion equation of the nanobeam by substituting Eq. (12) into Eq. (9)

$$\frac{\partial^4 w}{\partial x^4} - \beta_1 \frac{\partial^2 w}{\partial x^2} + \beta_2 \frac{\partial^2}{\partial t^2}\left(\mathrm{w} - \xi \frac{\partial^2 w}{\partial x^2}\right) + \beta_3 w + \beta_4 \frac{\partial^2 M_T}{\partial x^2} + \beta_5 \left(\xi \frac{\partial^2}{\partial x^2} - 1\right) q(x) = 0, \tag{13}$$

where

$$\beta_1 = \frac{\xi K_w + K_s}{IE + \xi K_s}, \quad \beta_2 = \frac{\rho A}{IE + \xi \quad s}, \quad \beta_3 = \frac{K_w}{IE + \xi K_s}, \quad \beta_4 = \frac{\alpha_{tt}}{IE + \xi \quad s}, \quad \beta_5 = \frac{1}{IE + \xi K_s}. \tag{14}$$

Substituting Eq. (5) into Eq. (2), gives the generalized heat conduction equation without the heat source ($Q = 0$), as

$$\left(1 + \tau_\theta \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial z^2}\right) = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left[\frac{\rho C_E}{K}\frac{\partial \theta}{\partial t} - \frac{\gamma T_0}{\mathrm{K}} z \frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right]. \tag{15}$$

4. The technique of the solution

We assume that the nanobeam has thermal insulation, thus $\partial\theta/\partial z$ at the upper and bottom surfaces of the nanobeam ($z = \pm h/2$) should have no effect at all. Additionally, we consider the increment temperature to vary in a sinusoidal fashion along the direction thickness as

$$\theta(x,z,t) = \Theta(x,t)\sin\left(\frac{\pi z}{h}\right). \tag{16}$$

When Eq. (16) is substituted into Eq. (13), the motion equation of the nanobeams is obtained as

$$\left(\frac{\partial^4}{\partial x^4} - \beta_1 \frac{\partial^2}{\partial x^2} + \beta_2 \frac{\partial^2}{\partial t^2}\left(1 - \xi \frac{\partial^2}{\partial x^2}\right) + \beta_3\right) w + \frac{24\beta_4}{h\pi^2}\frac{\partial^2 \Theta}{\partial x^2} + \beta_5\left(\xi \frac{\partial^2}{\partial x^2} - 1\right) q(x,t) = 0. \tag{17}$$

The flexure moment can also be derived using Eqs. (12) and (16) as

$$M(x,t) = \xi A\rho \frac{\partial^2 w}{\partial t^2} + \xi K_w w(x,t) - (IE + \xi K_s)\frac{\partial^2 w(x,t)}{\partial x^2} - \frac{24\alpha_t}{h\pi^2}\Theta. \tag{18}$$

Now, the generalized thermal conductivity equation can be derived from the Eqs. (15) and (16) on the form

$$\left(1 + \tau_\theta \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 \Theta}{\partial x^2} - \frac{\pi^2}{h^2}\Theta\right) = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left[\frac{\rho C_E}{K}\frac{\partial \Theta}{\partial t} - \frac{\gamma T_0 \pi^2 h}{24\mathrm{K}}\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right]. \tag{19}$$

To enable the numerical analysis more straightforward, dimensionless parameters are introduced . We'll utilize the following non-dimensional variables

$$\{x', z', u', w', h'\} = \frac{1}{\mathrm{L}}\{x, z, u, w, h\}, \quad \{t', \tau'_0, \tau'_\theta, \tau'_q\} = \frac{c_0}{\mathrm{L}}\{t, \tau_0, \tau_\theta, \tau_q\},$$
$$\xi' = \frac{\xi}{L^2}, \quad \Theta' = \frac{\Theta}{T_0}, \quad c_0\mathrm{L} = \frac{K}{\rho C_E}, \quad M' = \frac{M}{ALE}, \quad c_0 = \sqrt{\frac{E}{\rho}}, \quad q' = \frac{A}{EI}q. \tag{20}$$

Therefore, the essential Eqs. [17-19] in its new versions are

$$\frac{\partial^4 w}{\partial x^4} - A_1 \frac{\partial^2 w}{\partial x^2} + A_2 \left(\frac{\partial^2 w}{\partial t^2} - \xi \frac{\partial^4 w}{\partial t^2 \partial x^2} \right) + A_3 w = -A_4 \frac{\partial^2 \Theta}{\partial x^2} - A_5 \left(\xi \frac{\partial^2}{\partial x^2} - 1 \right) q(x), \quad (21)$$

$$M(x,t) = \xi \frac{\partial^2 w}{\partial t^2} + A_6 w(x,t) - A_7 \frac{\partial^2 w(x,t)}{\partial x^2} - A_8 \Theta - A_9 q(x), \quad (22)$$

$$\left(1 + \tau_\theta \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 \Theta}{\partial x^2} - \frac{\pi^2}{h^2} \Theta \right) = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2!} \frac{\partial^2}{\partial t^2}\right) \left[\frac{\partial \Theta}{\partial t} - \frac{\gamma \pi^2 h c_0 L}{24\mathrm{K}} \frac{\partial}{\partial t} \left(\frac{\partial^2 w}{\partial x^2} \right) \right], \quad (23)$$

where

$$A_1 = L^2 \beta_1, \quad A_2 = L^2 \beta_2 c_0^2, \quad A_3 = L^4 \beta_3, \quad A_4 = \frac{24 \ {}_4 T_0}{h \pi^2}, \quad A_5 = \frac{\beta_5 EI L^3}{A},$$

$$A_6 = \frac{\xi L K_w}{AE}, \qquad A_7 = \frac{IE + L^2 \xi K_s}{AEL^2}, \qquad A_8 = \frac{24 \ {}_0\alpha_{tt}}{AE \ {}^2 h \pi^2}, \quad A_9 = \frac{\xi L \mathrm{I}}{A^2}. \quad (24)$$

Prime sign have been removed for ease of use.

The external load $q(x,t)$ is supposed to be concentrated and moving at a constant speed υ along the axis of the beam. The load $q(x,t)$ can therefore be represented as

$$q(x,t) = Q_0 \delta(x - \upsilon t), \quad (25)$$

where Q_0 is the strength of load which assumed to be constant and $\delta(\cdot)$ is the Dirac function.

5. Boundary/Initial conditions

The boundary and initial conditions must be taken into account in order to get the sol ution. The initial conditions are assumed to be homogenous as follows

$$\Theta(x,0) = \frac{\partial \Theta(x,0)}{\partial t} = 0 = w(x,0) = \frac{\partial \mathrm{w}(x,0)}{\partial t}. \quad (26)$$

We shall assume that the nanobeam's two endpoints are clamped, i.e.

$$\mathrm{w}(0,\mathrm{t}) = \mathrm{w}(L,\mathrm{t}) = 0 = \frac{\partial^2 \mathrm{w}(0,\mathrm{t})}{\partial x^2} = \frac{\partial^2 \mathrm{w}(L,\mathrm{t})}{\partial x^2}. \quad (27)$$

In addition, we assume the nanobeam to be thermally loaded by ramp-type heating, which result s in

$$\Theta(x,t) = \Theta_0 \begin{cases} 0, & t < 0, \\ \frac{t}{t_0}, & 0 \le t \le t_0 \\ 1, & t > t_0 \end{cases}, \quad (28)$$

where Θ_0 is a constant and the ramp-type parameter t_0 is non-negative parameter. Additionally, the temperature at the end boundary needs to adhere to the relationship below

$$\frac{\partial \Theta}{\partial x} = 0 \quad \text{on} \quad x = L.$$

(29)

6. The effects of Laplace transformation

The Laplace transformation approach may be used to solve the governing and constitutive equat ions in closed form. By applying the Laplace transformation that is defined in the following form

$$\bar{f}(x,s)=\int_0^\infty f(x,t)e^{-st}dt. \tag{30}$$

on the homogeneous initial conditions (26), and to the both sides of Eqs. (21)-(23), it yields the new form of theses equations as following

$$\left(\frac{d^4}{dx^4}-A_{10}\frac{d^2}{dx^2}+A_{11}\right)\bar{w}=-A_4\frac{d^2\bar{\Theta}}{dx^2}+A_5\bar{g}(s)e^{-\frac{s}{v}x}Q_0, \tag{31}$$

$$\bar{M}(x,t)=A_{12}\bar{w}-A_7\frac{d^2\bar{w}}{dx^2}-A_8\bar{\Theta}-A_{13}\bar{g}(s)e^{-\frac{s}{v}x}Q_0, \tag{32}$$

$$\left(\frac{d^2}{dx^2}-B_1\right)\bar{\Theta}=-B_2\frac{d^2\bar{w}}{dx^2}, \tag{33}$$

where

$$A_{10}=(A_1+s^2\xi A_2),\ A_{11}=(A_3+s^2A_2),\ A_{12}=(\xi s^2+A_6),\ A_{13}=\frac{A_9}{1-\xi\frac{s^2}{v^2}},$$
$$A_{14}=\frac{\pi^2}{h^2}\ A_{15}=\frac{\gamma\pi^2hc_0L}{24K},\quad B_1=A_{14}+\frac{s(1+\tau_q s+\frac{1}{2!}\tau_q^2s^2)}{1+\tau_\theta s},\ B_2=\frac{sA_{15}(1+\tau_q s+\frac{1}{2!}\tau_q^2s^2)}{1+\tau_\theta s}. \tag{34}$$

The following differential equation for $\bar{w}$ is obtained by removing the function $\bar{\Theta}$ from from Eqs. (31) and (33) as

$$(D^6-AD^4+BD^2-C)\bar{w}(x)=\Gamma_1Q_0e^{-\frac{s}{v}x},$$

(35)

where the coefficients A, B and C are given by

$$A=A_4B_2+A_{10}+B_1,\quad B=A_{11}+B_1A_{10},\ C=B_1A_{11},\ D=\frac{d}{dx}, \tag{36}$$
$$\Gamma_1=A_5\left(\frac{s^2}{v^2}-B_1\right)\bar{g}(S).$$

Equation (35) can be moderated to

$$(D^2-m_1^2)(D^2-m_2^2)(D^2-m_3^2)\bar{w}(x)=\Gamma_1Q_0e^{-\frac{s}{v}x}, \tag{37}$$

where m_n^2, $n=1,2,3$ are roots of the characteristic equation

$$m^6-Am^4+Bm^2-C=0. \tag{38}$$

The general solutions of $\bar{w}$ may be obtained from equation (37) as following

$$\bar{w}=\sum_{i=1}^{3}(C_ie^{-m_ix}+C_{i+3}e^{m_ix})+C_7e^{-sx/v}. \tag{39}$$

where

$$C_7 = \frac{\Gamma_1}{(s/\upsilon)^6 - A(s/\upsilon)^4 + B(s/\upsilon)^2 - C}\ . \tag{40}$$

With likewise eliminating $\bar{\Theta}$ between (31) and (32), we get at the equation below, which is fulfilled by

$$\left[\frac{d^6}{dx^6} - A\frac{d^4}{dx^4} + B\frac{d^2}{dx^2} - C\right]\bar{\Theta} = \Gamma_2 Q_0 e^{-\frac{s}{\upsilon}x},$$

(41)

where,

$$\Gamma_2 = -s^2 A_5 B_2 \bar{g}(s)/\upsilon^2.$$

(42)

Eq. (41) generic solutions can be simplified as follows

$$\bar{\Theta} = \sum_{i=1}^{3}(F_i e^{-m_i x} + F_{i+3}e^{m_i x}) + C_8 e^{-sx/\upsilon}. \tag{43}$$

Substituting the functions $\bar{w}$ and $\bar{\Theta}$ from Eqs. (39) and (43) into Eq. (33) yields

$$F_i = \beta_i C_i, \quad F_{i+3} = \beta_i C_{i+3}, \qquad \beta_i = -\frac{B_2 m_i^2}{m_i^2 - B_1}, \quad C_8 = -\frac{B_2 s^2/\upsilon^2}{s^2/\upsilon^2 - B_1}C_7, \tag{44}$$

So,

$$\bar{\Theta} = \beta_i \bar{w} = \sum_{i=1}^{3}\beta_i(C_i e^{-m_i x} + C_{i+3}e^{m_i x}) + C_8 e^{-sx/\upsilon}. \tag{45}$$

By the same way we can arrive to the following equation for the bending moment $\bar{M}$ in the form

$$\bar{M} = \sum_{i=1}^{3}(-A_7 m_i^2 - A_2\beta_i + A_{12})\,(C_i e^{-m_i x} + C_{i+3}e^{m_i x}) + C_9 e^{-\frac{s}{\upsilon}x}, \tag{46}$$

where

$$C_9 = (-(s/\upsilon)^2 A_7 + A_{12})\,C_7 - A_8 C_8 - A_{13}\,Q_0\bar{g}(s). \tag{47}$$

Furthermore, the axial displacement after using Eq. (39) has the form

$$\bar{u} = -z\frac{d\bar{w}}{dx} = z\left[\sum_{i=1}^{3} m_i(C_i e^{-m_i x} + C_{i+3}e^{m_i x}) + (s/\upsilon)^2 C_7 e^{-sx/\upsilon}\right]. \tag{48}$$

Additionally, the strain will be

$$\bar{e} = \frac{d\bar{u}}{dx} = -z\left[\sum_{i=1}^{3} m_i^2(C_i e^{-m_i x} + C_{i+3}e^{m_i x}) + (s/\upsilon)^2 C_7 e^{-sx/\upsilon}\right]. \tag{49}$$

In the Laplace transform domain, boundary conditions (26) to (29) reduce to

$$\bar{w}(x,s)|_{x=0,L} = 0, \left.\frac{d^2\bar{w}(x,s)}{dx^2}\right|_{x=0,L} = 0, \tag{50}$$

$$\bar{\Theta}(x,s)|_{x=0} = \theta_0\left(\frac{1-e^{-t_0 s}}{t_0 s^2}\right) = \bar{G}(s), \tag{51}$$

$$\frac{\partial\bar{\Theta}}{\partial x} = 0, x = L. \tag{52}$$

The boundary conditions in Eqs. (39) and (45) are satisfied, according to Eqs. (50)- (52). Eqs. (37) and (45) are substituted into Eqs. (50)- (52) to provide

$$\sum_{i=1}^{3}(C_i + C_{i+3}) = -C_7, \tag{53}$$

$$\sum_{i=1}^{3}(C_i e^{-m_i L} + C_{i+3} e^{m_i L}) = -C_7 e^{-sL/v}, \tag{54}$$

$$\sum_{i=1}^{3} m_i^2 (C_i + C_{i+3}) = -C_7 s^2/v^2, \tag{55}$$

$$\sum_{i=1}^{3} m_i^2 (C_i e^{m_i L} + C_{i+3} e^{m_i x}) = -C_7 (s^2/v^2) e^{-sL/v}, \tag{56}$$

$$\sum_{i=1}^{3} \beta_i (C_i + C_{i+3}) = \bar{G}(s) - C_8, \tag{57}$$

$$\sum_{i=1}^{3} \beta_i m_i (-C_i\, e^{-m_i L} + C_{i+3} e^{m_i L}) = C_8 (s/v) e^{-sL/v}. \tag{58}$$

The unknown constants C_i and C_{i+3} may be determined using Eqs. (53)- (58). The lateral vibration and temperature may then be calculated using Eqs. (39), (45) and (46). (12). Eq. (48) may be used to calculate the displacement, and Eq. (46) can be used to get the bending moment. The Laplace transform of the complex solutions for the investigated fields in Laplace transform space is challenging to obtain. As a result, in the part that follows, the data will be numerically examined using a technique based on the Fourier series expansion approach. Software called Mathematica was used to carry out the numerical calculations.

7. The inverse of Laplace transforms

Following that, we encounter a large number of lengthy and complex expressions while finding answers in the transformed domain; hence, we employ a numerical technique developed by Durbin [29] to reverse the direction of Laplace twists to obtain solutions in a physical field. In this approach, the inversion solutions may be derived by applying the following formula:

$$M(t) = (2\text{Re}\sum_{k=1}^{m}[M(c_1 + ic_2)\, e^{-ic_2 t}] + M(c_1))\ \frac{e^{c_1 t}}{2t_1}, \tag{59}$$

$$c_2 = \frac{\pi k}{t_1},$$

where m is an integer big enough to signify the truncated parts number in the infinite

Fourier series and should be chosen as such

$$e^{c_1 t}\, Re(e^{-i\pi m t/t_1} M(c_1 + i\pi m/t_1)) \leq \epsilon,$$

where ϵ is a very small positive value matching to the required degree of precision, and c_1 is a real integer larger than the total of the real parts of all the singularities.

3. Results and discussion

To demonstrate the dynamic displacement response of an elastic beam to a moving load, numerical calculations are made. Silicon (Si), a superb material utilized in resonant devices, has been selected as the material for the purpose of numerical analysis. The following are the physical material data for silicon:

$$\alpha_T = 2.59 \times 10^{-6} K^{-1}, \nu = 0.22, K = 156\text{W}/(\text{mK}), T_0 = 293K$$
$$E = 169\text{GPa}, \rho = 2330\text{kg}/\text{m}^3, C_E = 713\text{J}/(\text{kgK}),$$
$$t_0 = 0.1\text{sec}, L/ \ = 10, b/ \ = 0.5, L = 1, z = \ /3.$$

The change in temperature θ, the deflection w, and axial displacement u distributions are graphed versus x –direction for two cases.

First case is examined to investigate changes in dimensionless lateral deflection w, change in temperature θ, and the displacement u with various phases delays where $t_0 = 0.1$ and $v = 0.22$ is presumptive values. The suggested model is used to construct the curves predicted by three different thermoelasticity models, which may be seen in figures 2 through 4 as special states of the two-phase-lag model (DPL). By taking different values of the coefficients of the phase lag model (, τ_q and τ_θ), we can get different models for the theory of thermoelasticity. For example we can obtain the modified Tzou's model by assuming $\tau_q = 0.2, \tau_\theta = 0.1$, the Lord Shulman model (LS) as $\tau_q = 0.2, \tau_\theta = 0$ and of course by putting $\tau_q = 0, \tau_\theta = 0$ we will get the classical thermoelasticity theory (CTE). The results reveal that phase delay effects have a big impact on how physical quantities are distributed. The mechanical patterns demonstrate that waves move across the material at a limited speed. It is also clear that several theories operate essentially the same manner. Additionally, it is demonstrated that when compared to other non-classical models, the CTE model yields greater values for the field variables. And due to the unrealistic physical demands of the thermodynamic notion of thermoelasticity, a better theory of thermal conductivity has been created. In contrast to the conventional theory, the modified theory proposes that thermal waves move at a finite pace.

Fig. 2 depicts the differences in the deflection w, this diagram illustrates that all values of w begin and terminate at zero and satisfy the boundary conditions at $x = 0$ and L. The lateral deviation similarly peaks at a specific distance from the left edge and declines as the x- axis increases.

For the change in temperature distribution θ which is shown in fig. 3, it's indeed obvious that the nonlocal stress appears to have minimal impact on the nanobeam's temperature change. The results also demonstrate that the temperature rapidly drops as x becomes farther and farther away from the thermal heat source. Indeed, contrary to what was predicted by conventional theories, actual findings have shown that size effects have a significant impact on the physical properties and mechanical responses of NEMS at the nanoscale at, defying the predictions of general theory [30–32].

According to the diagrams of u that is shown in fig. 4, it is noticed that all curves start at its maximum value and then decreases regularly to intersect the curves with the x -axis and then take negative values so that the wave rises again to reach zero again and settles at this position.

The second case investigates how changes deflection w, change in temperature θ, the displacement u, and the axial stress σ_{xx} at different axial speeds ($v = 2, 4, \text{and } 6$) are affected by sinusoidal heat pulses. The influence of velocity distribution v on the thermomechanical behavior of the nanobeams is shown in Figures 5–8. Any changes in the traveling speed will have an impact on the field amounts. As the values of axial speed v of grow, the magnitudes of mechanical waves w and u decrease. On the other hand, the axial speed v variations have a relatively small impact on the temperature change θ and the axial stress σ_{xx}.

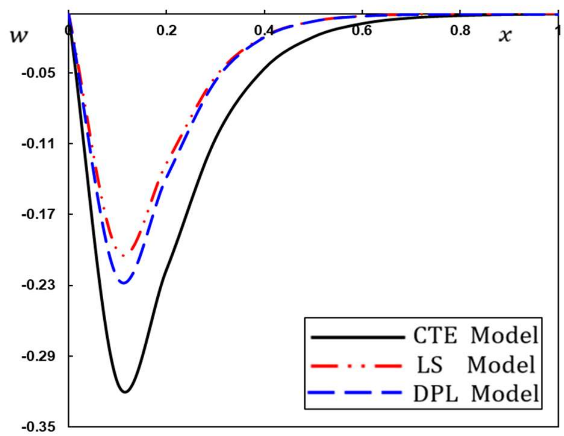

Fig. 2. The non-dimension lateral deflection w for various thermoelasticity models

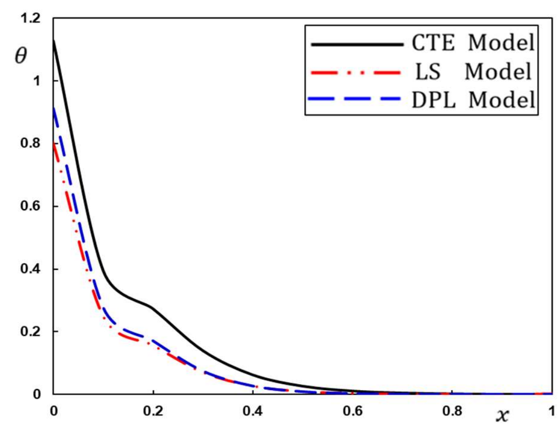

Fig. 3. The non-dimension temperature θ for various thermoelasticity models

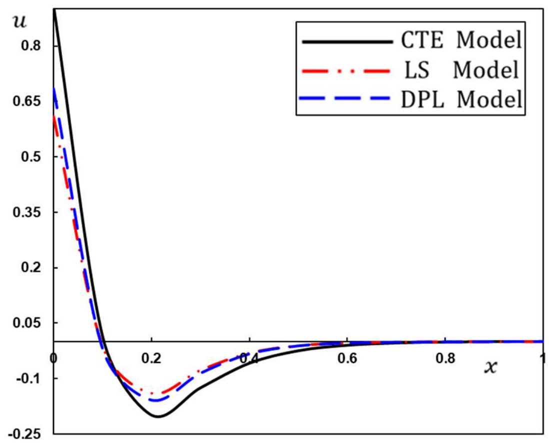

Fig. 4. The non-dimension axial displacement u for various thermoelasticity models

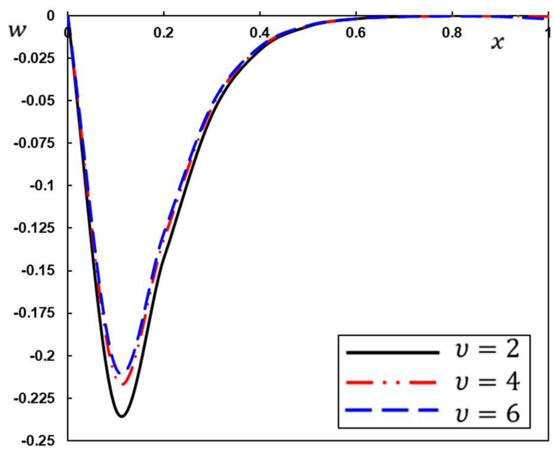

Fig. 5. The non-dimension deflection w for different values of the velocity υ

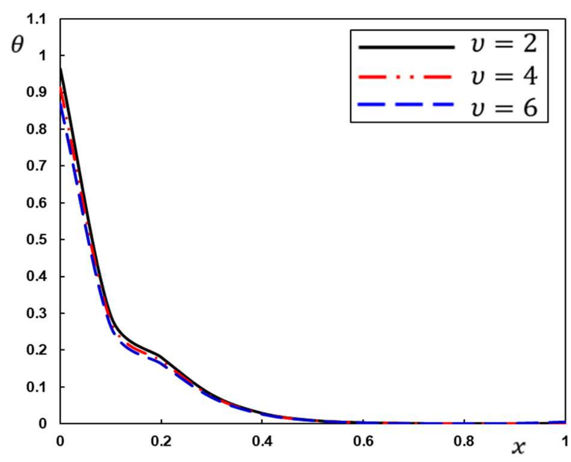

Fig. 6. The non- dimension temperature θ for different values of the velocity υ

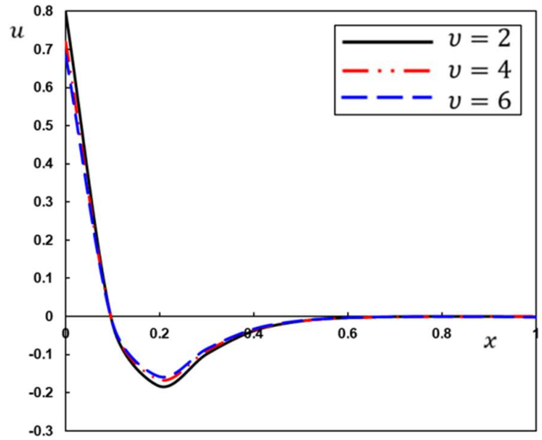

Fig. 7. The non-dimension axial displacement u for different values of the velocity υ

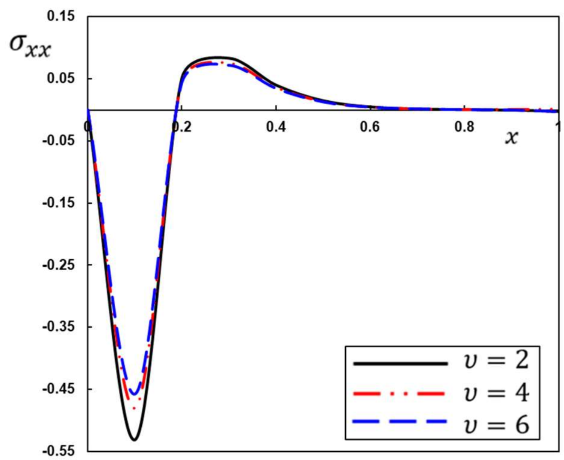

Fig. 8. The non-dimension axial stress σ_{xx} for different values of the velocity υ

Conclusion:

The governing equations of nonlocal nanobeams embedded in a two-parameter foundation and exposed to transverse moving load are built in the current study using extended thermoelasticity with phase lag theories and non-local Euler-Bernoulli beams. The two-parameter Pasternak foundation is used to represent the elastic foundation. Additionally, the discussion and investigation of the thermoelastic vibration of the temperature, deflection, displacement, and bending moment of nanobeam exposed to ramp-type heating.

On all the field variables, the impacts of the nonlocal parameter, elastic coefficient of the foundation, and shear layer foundation stiffness parameters have been demonstrated and graphically represented. The Pasternak foundation becomes a Winkler foundation if the stiffness of the shear layer foundation is disregarded.

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