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The linear Operators for Partial Ordered Banach Spaces

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Abstract:
In this study, we endeavour to prove that $K(X, Y)$is a structure and both sufficient to R.D.P .I will check that $L(X, Y)$ is conditionally complete or can contain a positive cone. I'm trying to find the conditions that make $K(X, Y)$ a network that has an approximate standard of unity.

Key Words: anach Space, Linear Operators, Normed Vector Spaces, R.D.P.

1. Introduction:

The connection between partial orderings on a Banach space and dual orderings on the dual space is generally recognized. The natural ordering of finite linear operators within partially ordered Banach spaces, however, seems to be extremely little understood. We examine the requirements on X and Y to ensure that $L(X, Y)$ is either conditionally complete, positively produced, or has a normal positive cone. Ellis ([1]) is credited with generating this space in a positive manner, which is uncommon. $L(X, Y)$ is order-unit-normed in this scenario, which happens when X is base-normed and Y is order unit-normed. We demonstrate that this outcome cannot be enhanced by permitting Y to be approximately –order uninformed, for instance. We deal with compact operators when we restrict our analysis to the scenario where Y is a simplex space. We determine the circumstances under which K(X, Y) must possess the Riesz decomposition property, be positively produced, or have a normal positive cone. We also establish conditions under which K(X, Y) is a lattice on X and Y Lastly, we demonstrate that if and only if X is base-normed is K(X, Y) approximate-order unit-normed.

2. Definitions and duality:

X_+ is a non-empty subset that: ?
(1) $X_+ + X_+ \subseteq X_+$, and

(2) $\lambda X_+ \subseteq X_+$ if $\lambda \geq 0$. is a segment in the real vector domain X.

(3) $X_+ \cap (-X_+) = \{0\}$as well, then $X_+$is referred to as a cone. We shall always assume that $X_+$is closed if X is a Banach space. If there is a neighborhood base of 0 in X, which is made up of sets U such that $x, z \in U$ and $x \leq y \leq z$ together imply that $y \in U$., then in this situation, $X_+$is called normal. It is obvious thatX_+ is a cone if it is normal. If each x in X can be expressed as $x^+ - x^-$,

where $x^+, x^- \in X_+$, then X_+ is generating. $X_+$is bounded generating if it is generating, which means that if M>0, then any $x \in X$

has a decomposition into positive components with $\|x^+\|, \|x^-\| \leq M\|x\|$. This is because X is a Banach space. We can define a partial preordering on X if and only if $x - y \in X_+$, provided that

X_+ is a wedge within X. This is a correct partial order if $X_+$is a cone, meaning that $x \geq y$ and $y \geq x$ implies $x = y$. Let X^* represent X's Banach dual. The dual wedge, denoted as

$$X_+^* = \{f_j \in X^*: \sum_j f_j(x) \geq 0 (x \in X_+)\}$$

, is a wedge in X^* The duality of the normalcy and producing qualities is among the most significant duality results. It is possible to make this quite exact. If $\|x\|, \|z\| \leq 1$ and $x \leq y \leq z$ imply that $\|y\| \leq C$, then call x^+ C-normal. It is also C-generating if, for every $x \in X$, there exist $x^+, x^- \in X_+$, with $x = x^+ - x^-$and $\|x^+\| + \|x^-\| \leq C\|x\|$. Next, we have:

Theorem (2.1): states that if and only if X_+^* is C-generating, then X_+ is C-normal. For all $\varepsilon > 0$, $X_+$is $(C + \epsilon)$-generating if and only if $X_+^*$is C-normal. Grosberg and Krein [2] are credited for the first portion of this result, and Ellis [3] for the second. Ng provides a brief proof in [4]. Ellis's result is connected to a number of other results. Here are a handful of them. Asimov states that if $x_1, x_2, \dots, x_n \geq 0$ implies $\sum_1^n \|x_i\| \leq (\mu + 1)\|\sum_1^n x_i\|$.

Xthen X is $(\mu + 1, n)$-additive. If $\|x_i\| \leq 1 (1 \leq i \leq n)$ indicates that there exists $y \geq x_1, \dots, x_n n$,with$\|y\| \leq \mu + 1$,then X is said to be $(\mu + 1, n)$-directed. If, on the other hand, X is $(\mu + 1 + \varepsilon, n)$ $-$directed for all $\varepsilon > 0$, then it is approximately $(\mu + 1, n)$-directed. In [5], the following theorem was demonstrated.

Theorem (2.2): states that if and only if X^* is $(\mu + 1, n)$ additive, then X^* is roughly $(\mu + 1, n)$ directed .Ng demonstrated the next two findings in [6].

Let $\mu \geq 0$. Apply the

Theorem (2.3): Then the following claims are interchangeable.

(i) $f_j, g_j \in X^*$ and $0 \leq g_j \leq f_j \Longrightarrow \sum_j \|g_j\| \leq (\mu + 1) \sum_j \|f_j\|$.

(ii) $x \in X$ and $\|x\| < 1 \Longrightarrow$ there exists $y \in X$ with $\|y\| < \mu + 1$ such that: $y \geq 0, x$.

Theorem (2.4): Let α≥1. Apply the Theorem (2.4). Then the following claims are interchangeable.

(i) $f_j, g_j \in X^*$ and $-f_j \leq g_j \leq f_j \Longrightarrow \sum_j \|g_j\| \leq (\mu + 1) \sum_j \|f_j\|$

(ii) $x \in X$ and $\|x\| < 1 \Longrightarrow$ there exists $y \in X$ with $\|y\| < \mu + 1$. such that $-y \leq x \leq y$. Ather kind of duality outcome relates to unique areas. A convex subset B of $X_+$is a base for X_+ if for every nonzero x∈ X_+, there exists a unique representation λb ,for $b \in B$, λ>0. If X is positively generated, the Minkowski functional of co(B∪-B) defines a semi norm on X. If this norm exists in the space X, it is referred to as a base normed space. An order unit in X is denoted by $e \in X_+$when λ>0 for every $x \in X$ and with $\lambda e \geq x \geq -\lambda e$. X with this norm is called order-unit-normed when the Minkowski functional of $\{x : e \geq x \geq -e\}$ is a norm. If there exists $v \in \Lambda$ and $\mu > -1$ with $(\mu + 1)e_v \geq x \geq -(\mu + 1)e_v$. Then an approximate order unit in X is an upward directed set $\{e_\lambda : \lambda \in \Lambda\}$in X. X with this norm is called approximate-order-unit-normed if the Minkowski functional of $\{x : \text{there exists } \lambda \in \Lambda \text{ with } e_\lambda \geq x \geq e_\lambda\}$ is a norm.

Theorem (2.5): The ensuing claims are interchangeable.

(i) X^* is base-normed.

(ii) X is approximate-order-unit-normed.

(iii) X_+ is 1-normal and the open unit ball of X is directed upwards.

Our ultimate goal is to determine the order characteristics of the relevant spaces. Remember that X possesses the Riesz decomposition property (R.D.P.) if there are x_1, x_2such that $0 \leq x_i \leq y_i$and $x_1 + x_2 = x$,whenever $0 \leq y_1 y_2$ and $0 \leq x \leq y_1 + y_2$. Alternatively put, X possesses the Riesz separation property (R.S.P.) if $u \in X$ such that $x, y \leq u \leq v, w$.whenever$x, y \leq u \leq v, w$If there is a least upper Bound for any memorized subset of a vector lattice X then X is said to be complete.

Let X be a partially ordered Banach space with a closed, normal, and generating cone, as stated in.

Theorem (2.6): The following are therefore comparable.

(i) X has the R.D.P.

(ii) X^* has the R.D.P.

(iii) X^* is a vector lattice.

(iv) X^* is a complete vector lattice.

(i) $\Longrightarrow$ (ii). This is well known.

(iii) $\Longrightarrow$ (iv). Assume that a subset of X_*, majorized by $h_0$is $\{f_i^j : i \in I\}$. $U(h) = \{g_i : h \geq g_i \geq \sum_j f_i^j, i \in I\}$, let. There is no empty set in $U(h)$ If $h \geq \sum_j f_i^j$ $(i \in I)$.According to the cone's definition in X_*, $U(h)$ is weak*-closed. Also take note of the fact that $U(h)$ is norm-bounded since X_+* is normal and X is positively produced.$U(h)$ is hence weak*-compact. Assume:

$U = \bigcap\{U(h) : h \geq \sum_j f_i^j\}$. The family $(U(h))_{h \geq \sum_j f_i^j}$ possesses the finite-intersection property because X* is a lattice. Given that $\{f_i^j\}$ is majorized, this family cannot be empty, so each $U(h)$ must be compact in order for U to be non-empty. For every i in I, $h \geq \sum_j f_i^j$ if h is in U.In contrast, $h \in U \subseteq Uh' \geq \sum_j f_i^j$ $(i \in I)$, meaning that $h' \geq h$. Thus, h is the supremum of $\{f_i^j : i \in I\}$.in X*.

(iv)$\Longrightarrow$(iii)$\Longrightarrow$(b).

This comes after a fortiori. Ando is responsible for the primary implication, (ii) $\Longrightarrow$ (i) [7].X and Y will be semi-ordered Banach spaces with closed cones throughout. The Banach space of all bounded linear operators with the standard norm from X to Y is denoted as $L(X,Y)$. This space will always be analyzed using the (closed) wedge $W = \{T : Tx \in Y_+\{x \in X_+)\}$. The subspace of all compact operators will be represented by $K\{X,Y)$, which has the same norm and wedge $K(X,Y) \cap W$. In order to rule out certain degenerate scenarios, we additionally presume that that $X_+, Y_+ \neq \{0\}$.

3. Bounded operators

The normalcy of $L(X,Y)_+$ is the first thing we address.

Theorem (3.1): If and only if X is positively produced and Y_+ is normal,

then $L(X,Y)_+$is normal. Assume that Y_= is D-normal and X is C-generated. Since $\|S\|, \|U\| \leq 1$. If $\|x\| \leq 1$, . Let $S, T, U \in L(X,Y)$, $x^+, x^- \geq 0$ such that $x = x^+ - x^-$ $\|x^+\| + \|x^-\| \leq C$. exist if ‖x‖≤1. When $\|Tx\| \leq \|Tx^+\| + \|Tx^-\|$, we observe that

$$\|T\| \leq \sup\{\|Tx^{+}\| + \|Tx^{-}\| : x^{+}, x^{-} \geq 0, \|x^{+}\| + \|x^{-}\| C\} \leq C \sup\{\|Ty\| : y \geq 0, \|y\| \leq 1\}$$
$$\leq C \sup\{\max\{\|Sy\|, \|Uy\|\} : y \geq 0, \|y\| \leq 1\}$$
$$\leq CD \max\{\|S\|, \|U\|\}.$$

On the other hand, let's say that $L(X,Y)_{+}$ is A-normal. Select $y \in Y_{+}$ such that $\|y\| = 1$.Fix $F: x \longmapsto f_j(x)y$ and so If $f_j, g_j, h_j \in X^{*}$ and$\sum_j f_j \leq \sum_j g_j \leq \sum_j h_j$. Given that $F \leq G \leq H$,$\|G\| \leq A \max\{\|F\|, \|H\|\}$. However, since $\sum_j \|f_j\| = \|F\|$ and so on, X_{+}^{*} is A-normal. X is hence positively created. allow f be a positive bounded linear functional on X, such that $f_j(x_0) = 1$ and allow $x_0 \in X_{+}(x \neq 0)$. Since X_{+} is closed, such f exists. Assume that $s, t, u \in Y$. Define S as $S :\longmapsto f_j(x)s$, and define T and U in the same way.$S :\longmapsto f_j(x)s$, $\|S\| = \sum_j \|f_j\| \|s\|$, and so forth are evident. Now, since $\sum_j \|f_j\| \neq 0$, Y_{+} must also be A-normal if $L(X,Y)_{+}$. Now, we examine $L(X,Y)$positive generation. It is rare to find situations where the space is positively generated. First, we examine a few prerequisites.

Proposition (3.2): states thatX_{+} is normal and Y is positively generated if $L(X,Y))$ is positively generated. Let f be a positive bounded linear functional on Y such that $f(y_0) = 1$and let $y \in Y_{+}$ with $\|y_0\| = 1$.let$G : x \longmapsto g_j(x)y_0$.if $g_j \in X^{*}$Given the positive generation of $L\ (X,Y)$, $H \geq G, 0$ exists, and $H \in L(X,Y)$. H is a positive bounded linear functional on X if $h(x) = f_j\big(G(x)\big)$ However, since $h \geq g_j$, so X^{*} is positively generated, and X_{+} is normal as a result. Let $x_0 \in X_{+}$ with $\|x_0\| = 1$.in order to demonstrate that Y is positively generated. Assume that f is a bounded linear functional on X such that :$\sum_j \|f_j\| = 1$. and $f_j(x_0) = 1$ Let $T : x \longmapsto f_j(x)y$. if y∈Y. We can find $S \geq T, 0$since $L(X,Y)$ is positively produced. Since $z \geq x, 0$ if $z = Sx_0$, Y is positively generated. If the closure of each open subset is open, then a compact Hausdorff space is stonean. When Y=C(Ω) and Ω is a stonean space, this is one of the situations in which $L(X,Y)$ is known to be positively generated whenever X_{+} is normal. Since C(Ω) is a full vector lattice in this situation (Nakano, [8]), we can benefit from the following theorem by Bonsall ([9]).

Theorem (3.3): Assume that E is a real vector space and that E_{+} represents a wedge in it. Assume that Q is a super linear map from E_{+}into V and that P is a sublinear map from E into a full vector lattice V such that $Q(x) \leq P(x))$ for any x in E_{+}Next, there is a linear operator T that maps from E into V in the following ways:

$$T(x) \leq P(x) \quad (x \in E),$$

$$Q(x) \leq T(x) \quad (x \in E).$$

First, we proved the dual of an Asimov [5] result for $Y = R$, which was asserted by Ng [28] without any supporting evidence.

Theorem (3.4): Let Ω be a stonean space. Then, if and only if $L(X, C(\Omega))$ is (a, undirected, then X is $(\mu + 1, n)$-additive. Assume that $T_1, \ldots, T_n : X \to C(\Omega)$ all have norms that are less than or equal to 1. In the event that $x \geq 0452142$. Let $Q(x) = \sup\{T_1(x_1) + 24 \cdots + T_n(x_n) : \sum_{i=1}^{n} x_i = x, x_i \geq 0\}$ Given that is clearly defined

$$\sum_{1}^{n} T_i(x_i) = \sum_{1}^{n} \|T_i(x_i)\| \, 1_\Omega \leq \sum_{1}^{n} \|x_i\| \, 1_\Omega \leq \alpha \left\| \sum_{1}^{n} x_i \right\| 1_\Omega \leq \alpha \|x\| 1_\Omega$$

and Q is super linearity on X_+ is evident. Moreover, $Q(x) \leq P(x)$ for any $x \in X_+$ if P is the sublinear map defined on X mapping $x \ to \ (\mu + 1)\|x\| \, 1_\Omega$. A linear operator S from X to $C(\Omega)$such that $S(x) \geq Q(x) \ (x \in X_+), , S(x) \leq P(x) \ (x \in X)$exists by Theorem (3.3).

The formulation of Q makes it obvious that if $X \geq 0$,and $1 \leq i \leq n$, then $S(x) \geq T_i$ (x).The implication in one direction is proven as ‖S‖≤ $\mu + 1$. Conversely, let's say that $x_1, \ldots, x_n \in X_+$.

Assume that $f_i \in X^*$ and that(x_i)=‖ x_i ‖.There is $S \geq T_i \ (1 \leq i \leq n)$, if $T_i : x \longmapsto f_i(x) 1_\Omega$, and $\|S\| \leq \mu + 1$. Nevertheless,

$$(\mu + 1) \left\| \sum_{1}^{n} x_i \right\| 1_\Omega \geq S\left(\sum_{1}^{n} x_i\right) = \sum_{1}^{n} S(x_i) \geq \sum_{1}^{n} T_i(x_i) = \sum_{1}^{n} \|x_i\| \, 1_\Omega.$$

Thus, $X$is$(\mu + 1, n)$-additive $(\mu + 1)\|\sum_1^n x_i\| \geq \sum_1^n \|x_i\|$and X Similar outcomes to Theorems (2.3) and (2.4) can be stonean space. The following are equivalent. Demonstrated.

Theorem (3.5): Let Ω be a stonean space and $\mu \geq 0$These two are interchangeable, for each

(i) $x, y \in X$ and $0 \leq x \leq y \Longrightarrow \|x\| \leq (\mu + 1)\|y\|$.

(ii) $S \geq T, 0$ with $\|S\| \leq \mu + 1$ exists if $T \in L(X, C(\Omega))$ and $\|T\| \leq 1$

(ii)⟹(i). Bonsall's theorem (Theorem 3.3), which uses $P(x) = (\mu + 1)\|T\| \, \|x\| 1_\Omega$ and $Q(x) = \sup\{Ty : 0 \leq y \leq x\}$, yields this again. (ii) ⟹(i). To demonstrate this, first observe that if $C(\Omega) = R$, then (i) holds with X replaced by X^{**}.Since X_+ and $X \subset X^{**}$ are closed,

(i) holds (i.e., the original ordering on X matches the relative ordering as a subspace of X^{**}).Generally, pick $\omega \in \Omega$,let$F : x \longmapsto f_j(x)1_\Omega$if f∈ $f_j \in X^*$ and $\sum_j \|f_j\| \leq 1$.According to $G \geq F, 0$,where $\|G\| \leq \mu + 1$. If $g_j : x \longmapsto G(x)(\omega)$; then $g_j \in X^*$,$g_j \geq f_j, 0$,and $\sum_j \|g_j\| \leq \mu + 1$

are evident. As a result, (i) is true if R is substituted for $C(\Omega)$.

Proposition (3.6): Let Ω be a stonean space and $\mu \geq 0$. These two are

interchangeable. For each

(i) $x, y \in X$ and $-y \leq x \leq y \Rightarrow \|x\| \leq \mu + 1 \leq \|y\|$.

(ii) $T \in L(X, C(\Omega))$ and $\|T\| \leq 1$⟹ there exists $S \geq T, -T$where $\|S\| \leq \mu + 1$.

(i) ⟹ (ii).Using $P(x) = (\mu + 1)\|T\|\|x\|1_\Omega$ and

$Q(x) = \sup\{Ty : -x \leq y \leq x\}$., we apply Theorem (3.3).

(ii)⟹(i). The proof is nearly the same as the one for Theorem (3.5). When either X or Y is finite-dimensional, we can demonstrate that $L(X, Y)$

is positively generated.

Lemma (3.7): Given a finite dimensional real vector space X and a closed, generating cone X_+

every P_i induces a lattice ordering on X, and there exist closed generating cones P_1, P_2 such that $P_1 \subseteq X_+ \subseteq P_2$First, note that X_+ must be normal as X is finite-dimensional and $X_+$is closed. Moreover, X_+ 's interior is not empty, giving X an order unit. The basis B of $X_+$is the same as that of X^*, B is undoubtedly compact, and if it's w-dimensional, we may identify $n+1$, affinity independent extreme points in B. These points have an n-simplex, S, as their convex hull. When P_1 is the (closed) cone with base S, $P_1 \subseteq X_+$ and $P_1 - P_1 = X$.are evidently related. Identify$P_1^* \subseteq X_+^*$ closed, generating, And inducing a lattice ordering before attempting to identify P_2.

Let $P_2 = P_1^* \subseteq X_+^{**} = X_+$now, where X is identified by X^{**}).

P_2 induces a lattice ordering on X via Theorem (1.2.8). Since P_2 is obviously closed and generating, the outcome is finished.

Proposition(3.8): $L(X,Y)$ is positively produced if X_+ is normal and Y is finite-dimensional and positively generated. Positive generation of $L(X,Y)$ with natural ordering occurs if Y is given the order produced by P_1However, if $S \geq T, 0$ for this ordering, $S \geq T, 0$ for the initial ordering as well. As a result, $L(X,Y)$ generates positively as needed.

Theorem (3.9): States that $L(X,Y)$ is positively generated if X is finite-dimensional and Y is positively produced. Take X and the cone P_2 that contains X_+. This will provide $X_+ - X_+$,which is built according to in Lemma(1.2.15). let$x_1, \dots, x_n$reside on one extreme ray of P_2if $T : X \to Y$. Together, they create P_2.Since Y is positively generated, for$1 \leq i \leq n$, $y_i \geq Tx_i, 0$.Let $S_i = y_i$

Then, by linearity, extend S to $X_+ - X_+$ and, in any linear way, to the entirety of X.For every $x \in P_2$,we thus have $Sx \geq Tx, 0$ and S is bounded since X is finite-dimensional. However, whenever $x \in X_+$, $Sx \geq Tx, 0$ ensuring that $L(X,Y)$ is positively generated .If X is base-normed and Y is order-unit-normed, this is another scenario in which the space is known to be positively generated

(Ellis ([1])).$L(X,Y)$ is order-unit-normed in this instance. One may argue that $L(X,Y)$ would have an approximate-order-unit norm if Y were to be assumed to have just an approximate-order-unit norm. We'll discover that this isn't feasible. Actually, we have the idea that follows.

Proposition(3.10): States that the unit ball of B is bounded above if $L(X,Y)$ is positively produced whenever X is base-normed. Specifically, if $Y_+$is normal, then an order-unit norm is equal to the norm on Y. Assume$X_1 = Y \times \boldsymbol{R}$ Assign Give X_1 the cone with base $\{(y,1) : \|y\| \leq 1\}$, along with the base norm that goes

with it. As the ball of Y has an upper constraint of $2\pi(0)$Because if $\|y\|1$, then $\|-y\| \leq 1$ as well, and we have projection of $X - L$ onto Y, let $T \in L(X_1, Y)$.Assuming $S \geq T, 0$. there exists $S \in L(X_1, Y)$Examine the map π, which defines
$\pi y = S(y, 1)$, from the unit ball of Y into Y. Clearly, for every y in Y, $\pi y \geq y, 0$ with

$\|y\| \leq 1$. π is affine as well. We assert that the unit

$$2\pi(0) = \pi(y) + \pi(-y) \geq y + 0 = y.$$

In case Y_+ is C-normal, the following inclusions are present: Thus, in this situation, the order unit norm generated on Y by 2π(0) is identical to the original norm

$$\{y \in Y : \|y\| \leq 1\} \subseteq \{y \in Y : 2\pi(0) \geq y \geq -2\pi(0)\} \subseteq \{y \in Y : \|y\| \leq 2C\|\pi(0)\|\}$$

We also own the subsequent dual outcome.

Proposition (3.11):

Assume that if X is positively created, then $L(X, Y)$is also positively generated if Y is ordered by the unit of measurement. In that case, a base norm and the

norm in X are equal. The map map $\pi : L(X, Y^*) \to L(Y, X^*)$, defined by

$$\big((\pi T)(y)\big)(x) = (Tx)(y),$$

is used to illustrate this. is $L(X, Y^*)$ linear isometry onto $L(Y, X^*)$ (see, for example, [10]). This map is also an order-isomorphism since

$$\pi T \geq 0 \Leftrightarrow (\pi T)(y) \geq 0 \qquad (y \in Y_+)$$

$$\Leftrightarrow \big((\pi T)(y)\big)(x) \geq 0 \qquad (y \in Y_+, x \in X_+)$$

$$\Leftrightarrow (Tx)(y) \geq 0 \qquad (y \in Y_+, x \in X_+)$$

$$\Leftrightarrow Tx \geq 0 \qquad (x \in X_+)$$

$$\Leftrightarrow T \geq 0.$$

Assume for the moment that a base norm and the norm on X are not equal. In such case, X^* is not equal to a space with an order unit norm. As a result, $L(Y, X^*)$ is not positively produced in base-normed space Y. In other words, Y^* is order-unit-normed, as claimed, but $L(Y, X^*)$ is not positively generated. These final two findings can be summed up as follows.

Corollary (3.12): Let X, Y, and W be the classes of partially ordered Banach spaces with generating cones that are closed and normal. Let X_1 represent the class of all such spaces that correspond to base-normed spaces, and let Y_1 ,represent the class of such spaces that correspond to order-unit-normed spaces. Assume further that $X \supseteq X_1$ and $Y \supseteq Y_1$. $L(X, Y)$ is positively generated, If $X \in X_1$ and $Y \in Y_1$.

(ii) $X = X_1$ and $Y = Y_1$ if $L(X, Y)$ is positively generated whenever $X \in X$ and $Y \in Y$.After discussing the spaces Y such that $L(X, Y)$is positively produced whenever X_+ is normal, we wrap up our investigation of the positive generation of $L(X, Y)$.We restrict ourselves to the spaces with normal positive cone in order to have some representation of the concerned spaces. Any such Y is unquestionably identical to an order-unit-normed space by Proposition (3.10). We know that Y has this property if Y is either finite-dimensional or of the form $C(\Omega)$,where Ω is astonean space, then we know that Y has this feature. But there are additional areas Y like this. To see this, we make use of:

Example (3.13): Let S be a non-isolated point in an infinite stonean space, S. Given $t, u \in S$,

let $S_1 = S \cup \{t\} \cup \{u\}$.In order to make S_1 stonean, let $U \subseteq S_1$be open if and only if $U \cap S$ is open.

Given $Y = \{f_j \in C(S_1) : 2f_j(s) = f_j(t) + f_j(u)\}$,clarify. Y is not a lattice since s is not isolated, despite having the R.D.P. Here is how to define $P : C(S_1) \to Y$

$$(Pf_j)(x) = f_j(x) + f_j(t) + f_j(u) \quad (x \in S) = f_j(s) + f_j(t) + f_j(u) \quad (x = t \text{ or } x = u).$$

It is evident that P satisfies every requirement of the lemma, and $P(C(S_1)) \subseteq Y$. Thus, whenever $X_+$is $n, L(X, Y)$ is positively produced. If Ω is a compact Hausdorff space and Y is a space $C(\Omega)$, then Ω is probably stonean if $L(X, Y)$ is positively generated whenever X_+ is normal. If Ω is metrizable, we can demonstrate this fact, as we show below. Though we need evidence, it is likely that if Y is separable and Y_+ is normal, then Y is finite-dimensional if it possesses this characteristic. The example demonstrates that little can be demonstrated, at least in words that are now in use, in the absence of the separability assumption or the assumption that Y is a lattice. Presumably, there is a sense in which Y is 'close' to a space C(Ω) with Ω stonean.

Theorem(3.14): Assume that Y is a separable lattice with normal Y_+. Y is finite dimensional if $L(X,Y)$ is positively generated for all X with X_+ normal. It is known that Y is equal to a space $C(\Omega)$, where Ω is compact and amortizable. If Ω. is not finite, let $\omega_0 \in \Omega$ be in the closure of $\Omega\backslash\{\omega_0\}$. Define an open set sequence as follows.

Since $d(\omega_1,\omega_0) < 1$ and $\omega_1 \neq \omega_0$, let $U_1 = \left\{\omega \in \Omega : d(\omega,\omega_1) < \frac{1}{2}d(\omega_0,\omega_1)\right\}$. Select $\omega_{n+1} \neq \omega_0$,such that $d(\omega_{n+1},\omega_0) < \frac{1}{2}d(\omega_n,\omega_0)$, provided that $\omega_n$and U_n are specified. Let $U_{n+1} = \left\{\omega \in \Omega : d(\omega,\omega_{n+1}) < \frac{1}{2}d(\omega_0,\omega_{n+1})\right\}$. The non-empty, open, and disjoint sets U_k are readily apparent. For any k, $\omega_0 \in \bar{V}_k\backslash V_k$, and for $V_k = \bigcup_{n=0}^{\infty} U_{4kn+2^k-1}$ $(k = 0,1,2,\dots)$, the same holds true. Given $f_j \in C(\Omega)$ and χ_k the characteristic function of V_k, let X be the vector space of all bounded real-valued functions defined on Ω of the type

$$f_j + \sum_{0}^{\infty} \lambda_k \chi_k$$

(pointwise convergence). First, take note that no other element of X can be broken down into this form. This is because $f_j(\omega_0) = g_j(\omega_0)$)if g is the function and $g_j = f_j + \sum_0^\infty \lambda_k \chi_k$. Nevertheless, $f_j = g_j - \sum_0^\infty \lambda_k \chi_k$ and $\lambda_k = \lim\left(g_j(\omega) - f_j(\omega_0)\right)$ as $\omega \to \omega_0$ in V_k indicate that $f_j$and$\lambda_k$are properly defined. Given the supremum norm, it can be inferred that X is a Banach space due to the uniqueness of this decomposition. Yes, please $\sum_j g_n^j = \sum_j f_n^j + \sum_{k=0}^\infty \lambda_k \chi_k$,and assume that $\sum_j \left\| g_n^j - g_m^j \right\| < \varepsilon$. It follows that

$$\sum_j \left| g_n^j(\omega_0) - g_m^j(\omega_0) \right| < \varepsilon \qquad (1)$$

$$\lim \sum_j \left| g_n^j(\omega) - g_m^j(\omega) \right| < \varepsilon \quad (\text{as } \omega \to \omega_0 \text{ in } V_k) \qquad (2)$$

$$|\lambda_k^n - \lambda_k^m| < 2\varepsilon \tag{3}$$

and

$$\sum_j \|f_n^j - f_m^j\| < 3\varepsilon \tag{4}$$

Therefore, $\sum_j \left(f_n^j\right)_{n=1}^{\infty}and(\lambda_k^n)_{n=1}^{\infty}$ are also Cauchy sequences if $\sum_j \left(g_n^j\right)_{n=1}^{\infty}$ is. If the limits of these final two are f and λ k, respectively (in $C(\Omega)$) and R), then the limit of $\sum_j \left(g_n^j\right)_{n=1}^{\infty}$in X is $g_j = f_j + \sum_0^{\infty} \lambda_k \chi_k$. Assume that $T :\ g_j = f_j + \sum_0^{\infty} \lambda_k \chi_k \longmapsto f_j$ We are aware that T is unambiguously linear and properly defined. Moreover, T is bounded for, as we can see from (4) above, where $\sum_j g_n^j = \ g_j$ and ,$\sum_j g_m^j = 0$ and $\sum_j \|T g_j\| = \sum_j \|f_j\| \leq 3\|g_j\|$.

If $L(X, Y)$ with $S \geq T, 0$ would exist if $L(X, Y)$ were positively produced. We examine S1.

$$S1 \geq S\left(\sum_j g_n^j\right) = \sum_0^n \left(S_{\chi_k}\right). \tag{5}$$

holds for every n. Let $\sum_j f_{k,\omega}^j$ be any continuous function on Ω such that $0 \leq \sum_j f_{k,\omega}^j \leq \sum_j f_{k,\omega}^j \big|$E $V_k \equiv 0$,, and $\omega \in V_k$. The existence of such a function is established by Urysohn's lemma. As $S \geq T, 0$, we have $\chi_k \geq \sum_j f_{k,\omega}^j \geq 0$.

$$S_{\chi_k} \geq S \sum_j f_{k,\omega}^j \geq T \sum_j f_{k,\omega}^j = \sum_j f_{k,\omega}^j.$$

Specifically, $S_{\chi_k}(\omega) \geq 1$ in all cases where $\omega \in V_k . S_{\chi_k}(\omega_0) \geq 1$ since S_{χ_k} is continuous. We thus get $S_1(\omega_0) \geq n + 1$ for all n from (5). This is obviously not feasible, and the outcome is established. We examine briefly the order structure of $L(X, Y)$ in the case where X and Y have normal, closed generating cones. If there is a least upper bound for each subset that is bounded above, we will refer to a partially ordered Banach space as conditionally complete. We don't take the space to be a lattice.

Proposition(3.15): Let X and Y have a partial order. Cones that are closed and normal generate Banach spaces. The following are therefore comparable.

(i) The conditional completion of $L(X,Y)$ is met. (ii) Y is a full vector lattice and X possesses the R.D.P.

(ii) $\Longrightarrow$ (i). Assume that

$$T_{(\mu+1)} \leq T_0 \in L(X,Y).$$

and that $\{T_{(\mu+1)} : \mu+1 \in A\}$ is a subset of $L(X,Y)$.

If $x \in X_+$ and $x = \sum_1^n x_k$ with $x_k \in X_+$, then

$$\sum_{k=1}^{n} T_{(\mu+1)_k} x_k \leq \sum_{k=1}^{n} T_0 x_k = T_0 x$$

For each $(\mu+1)_k \in A$,
$\sum_{k=1}^n T_{(\mu+1)_k} x_k \leq \sum_{k=1}^n T_0 x_k = T_0 x$ Thus, the set's supremum

$$\left\{\sum_{k}^{n} T_{(\mu+1)_k} x_k : x = \sum_{k=1}^{n} x_k \, , x_k \in X_+, (\mu+1)_k \in A\right\}$$

existing in Y. Use Sx to indicate this. A classic argument states that the map

$S : x \longmapsto Sx$ is additive on the positive cone of X since X has the R.D.P. S can also be expanded into a linear operator that goes from X to Y. $S \geq T_{(\mu+1)}$, and U≥S if U is any linear operator from X to Y with $U \geq T_{(\mu+1)}$, are obvious conclusions. It is still to be proven that $S \in L(X,Y)$.

If $x \geq 0$, then Now $\left(T_0 - T_{(\mu+1)}\right)x \geq (T_0 - S)x \geq 0$ if $x \geq 0$. We can write each $x \in X$, with $\|x\| \leq 1$, as$x^+ - x^-$ where $x^+, x^- \in X_+$,, and

$\|x^+\| + \|x^-\| \leq C$ since X is C-generating for some $C > 0$. Next, we have

$$-x^- \leq x \leq x^+,$$

meaning that

$$-(T_0 - S)x^- \leq (T_0 - S)x \leq (T_0 - S)x^+.$$

Hence $T_0 - S \in L(X, Y)$. But $T_0 \in L(X, Y)$ by assumption, so $S \in L(X, Y)$.

However,

$$(T_0 - S)x^\pm \leq (T_0 - T_\alpha)x^\pm,$$

is also present, meaning that -

$$-\left(T_0 - T_{(\mu+1)_\alpha}\right)x^- \leq (T_0 - S)x \leq (T_0 - S)x^+.$$

We observe that

$$\|(T_0 - S)x\| \leq$$

$$P \max\{\left\|\left(T_0 - T_{(\mu+1)}\right)x^-\right\|, \left\|\left(T_0 - T_{(\mu+1)}\right)x^+\right\|\}$$

$$\leq CP\left\|T_0 - T_{(\mu+1)}\right\|.$$

since Y is P-normal for some $P > 0.T_0 - S \in L(X, Y)$thus resides in $L(X, Y)$. However, assuming $T_0 \in L(X, Y)$, $S \in L(X, Y)$.

(i) $\Longrightarrow$ (ii). It will be sufficient to demonstrate that Y is conditionally complete

because it is expected that Y is positively produced. allow g be a positive bounded linear functional on X such that $g_j(x_0) = 1$ and allow $x_0 \in X_+, \|x_0\| = 1$. Let $(s_i)_{i \in I}$ be a family in Y that has t as its upper bound. Define $S_i : x \longmapsto g(x)s_i$, then define T in the same way.$(T - S_i)x = (t - s_i)g_j(x) \geq 0$,, meaning that

$T \geq S_i$ .Assuming that T_0 exists, let it be the supremum of $(S_i)_{i \in I}$ in $L(X, Y)$.

$T_0 \geq S_i, T_0 x_0 \geq S_i x_0 = s_i$ since $T_0 \geq S_i$.Conversely, if $T_1 \geq S_i$, $t_1 \geq s_i$is defined as $T_1 : x \longmapsto g_j(x)t_1$Since $t_1 = T_1x_0 \geq T_0x_0$ .The supremum of $(s_i)_{i\in I}$ is therefore

T_0x_0.First, note that $L(X, Y)$ has the R.D.P. in order to demonstrate that X possesses it. With$(y_0) = 1$ for $y_0 \in Y_+$, let f be a bounded positive linear functional on Y. Define $G : x \longmapsto g_j(x)y_0$ if $g_j, h \geq m, n$ with all these elements of X^*, and so on. Then, $L(X, Y)$ includes $G, H \geq M, N$, and. Consequently, $L \in L(X, Y)$ exists where

$G, H \geq L \geq M, N$..When f_j is used to compose, it is evident that $f_j \circ L \in X^*$and that

$$g_j, h \geq f_j \circ L \geq m, n,$$

As a result, under Theorem (2.6), X has the R.D.P. since X^* has the R.D.P.

4. Compact operators

Given the findings of linden Strauss [11], it appears that the most practical range selection for a study of compact operators is those whose duals are $L^1(\alpha - 1)$ spaces. We will assume that the range is a simplex space as we are working with partially ordered spaces (Afros, [12] and [13]). Y will therefore represent a simplex space for the remaining values. This space has the supremum norm and natural partial order, and it is isometrically order isomorphic to a space of continuous affine functions on a compact simplex

that vanishes at one extreme point. Y may be identified with $A_0(K)$and $K = \{f_j \in Y^* : f_j \geq 0, \|f_j\| \leq 1\}$

is a compact simplex given the weak* topology (the distinguishing extreme point being 0). Obtain the following outcome.

Proposition (4.1): Assume that T is a bounded linear operator from X into $A_0(K)$. and that X is a Banach space. Then, for the weak* topology of X^*, there exists an affine map τ from K into X^*that vanishes at 0 and is continuous, so

(i) $(Tx)(k) = (\tau k)(x)$ $(x \in X, k \in K)$,

(ii) $\|T\| = \sup\{\|\tau k\| : k \in K\}$.

In contrast, (i) defines a bounded linear operator from X to $A_0(K)$ with norm described by (ii) if such a map τ is given. If and only if τ is continuous for the norm topology of X^*, then T is compact. $T \geq 0$ if and only if $\tau \geq 0$. If X is partially sorted by a closed cone. With the exception of the final comment, all of these claims are (mostly) supported by [14]. One may apply the same reasoning as in the Proposition (3.11) proof.

Let F be a Frechet space and K be a simplex. If the set

$\{k \in K : \Phi(k) \cap U \neq \emptyset\}$ is open in K whenever U⊆F is open, then a map $\Phi : K \to 2^F$ is called lower semi continuous. If Φ(k) is a nonempty convex Set ,and

$$\lambda\Phi(k_1) + (1-\lambda)\Phi(k_2) \subseteq \Phi(\lambda k_1 + (1-\lambda)k_2)$$

whenever $k_1 k_2 \in K$ and $0 \leq \lambda \leq 1$, then Φ is said to be affine. The subsequent Lazar theorem [15].

Theorem (4.2): Let $\Phi : K \to 2^F$ be an affine lower semi continuous map such that $\Phi(k)$ is closed

for all $k \in K$. Let F be a Fr'echet space. Then, for every k in K, there exists a continuous affine

map $\varphi : K \to F$ such that $\varphi(k) \in \Phi(k)$ via affine continuous selection for Φ.

Proposition (4.3): Assume that X is a partially ordered Banach space with a closed cone and that Y is a simplex space. For any $\varepsilon > 0$, X_+ is $(C+\varepsilon)$- We limit our analysis to The case where Y is a simplex space; in this instance, the positive generation of $K(X,Y)$ is satisfied.

Theorem (4.4): Assume that $\mu \geq 0$, X is a partially ordered Banach space with a closed cone. And

Y is a simplex space. These two are interchangeable. For every $x, y \in X$ and $0 \leq y \leq x \Longrightarrow \|y\| \leq (\mu+1)\|x\|$. If T is in the set $K(X,Y)$ and $\|T\| < 1$, then S is in the set $K(X,Y)$ and $S \geq T, 0$ and $\|S\| < \mu + 1$. Proposition (4.1) allows us to associate $A_0(K, X^*)$ with $K(X,Y)$We assume $\pi \in A_0(K, X^*)$and$\|\pi\| \leq 1$in

order to demonstrate that (i)⟹(ii). It is sufficient to demonstrate that $\varphi \in A_0(K, X^*)$with $\varphi \geq \pi, 0$and $\|\varphi\| \leq \mu + \varepsilon + 1$ exists for all $\varepsilon > 0$. Assume that $\Phi(0) = \{0\}$ and that

$\Phi(k) = \{e \in X^* : e \geq \pi(k), 0$ and $\|e\| < \mu + \varepsilon + 1\}^-$ $(k \neq 0)$. For every k in K, it is evident that $\Phi(k)$ is closed and convex. Furthermore, under Theorem (2.3), $\Phi(k)$ is non-empty. We demonstrate that Φ is lower

semi continuous and affine. According to Theorem (4.2), Φ will be selected in a continuous affine manner to yield the necessary φ. Let x belong to $\Phi(k)$,

x' to $\Phi(k')$, and $0 \leq \lambda \leq 1$.$\lambda x + (1-\lambda)x' \geq \pi(\lambda k + (1-\lambda)k'), 0$, is unquestionably true, and it is also readily apparent that the requirement that the points be limits of comparable points of norm strictly less than $\mu + \varepsilon + 1$ is satisfied. Consequently, we may observe that

$$\lambda\Phi(k) + (1-\lambda)\Phi(k') \subseteq \Phi(\lambda k + (1-\lambda)k').$$

if $\lambda k + (1-\lambda)k' \neq 0$.However, as 0 is an extreme point of K, $\lambda k + (1-\lambda)k' = 0$, so the inclusion is still valid. Assume $k_0 \in K$ if, $D \subseteq E$ is open, and $\Phi(k_0) \cap D \neq \emptyset$.It is obvious that we may get

$$d_0 \in \Phi(k_0) \cap D$$

With $\|d_0\| < \mu + \varepsilon + 1$. Assume : $\{d: \|d - d_0\| < \eta\} \subseteq D$ and let: $\|d_0\| = (\mu + \varepsilon + 1) - \delta$.

The open set

$$\left\{k \in K: \|\pi(k) - \pi(k_0)\| < \frac{1}{2}\min\{\eta, \delta\}/\mu + 1\right\}$$

is represented by U.If $k \in U$, then we can find $p \geq \pi(k) - \pi(k_0), 0$, with

$\|p\| \leq (\mu + 1)\|\pi(k) - \pi(k_0)\| \leq \frac{1}{2}\min\{\eta, \delta\}$,Since $d = d_0 + p \geq \pi(k), 0$

and $d \in \Phi(k)$ then $\|d\| \leq \|d_0\| + \|p\| \leq \mu + \varepsilon - \frac{1}{2}\delta + 1$.Additionally, $d \in D$

since $\|d - d_0\| \leq \frac{1}{2}\eta$. Now, it is unquestionably true that $\Phi(k) \cap D \neq \emptyset$ if

$k \in U\backslash\{0\}(k \in U$ if $k_0 = 0)$and that Φ is lower semi continuous.

Select $y_0 \in Y_+$with$\|y_0\| = 1$,and$\sum_j f_0^j \in Y_+^*$with $\sum_j \|f_0^j\| = \sum_j |f_0^j(y_0)| = 1$

in order to demonstrate the opposite. (because Y is a simplex space, perhaps). Assume g_j is in X^* and $\sum_j \|g_j\| \leq 1$.In $K(X,Y)$, $G : x \mapsto g_j(x)y_0$, and

$\|G\| = \sum_j \|g_j\|$.There is $H \in K(X,Y)$ with $H \geq G, 0$ and $\|H\| \leq \mu + \varepsilon + 1$ for

any $\varepsilon > 0$.We obtain $h \geq g_j, 0$,, and $\|h\| \leq \mu + \varepsilon + 1$ by putting

$h(x) = \sum_j f_0^j\,(Hx)$.

We now know that (i) holds according to Theorem (3.5). The space $A(K,E)$ is positively generated if and only if E is positively generated, as demonstrated by Asimov and Atkinson ([1]). Additionally, they demonstrate that if $E_+$is closed, normal, and generating, then $A(K,E)$has the R.D.P. if and only if E possesses this quality. This outcome becomes necessary only if E is a lattice, and in that instance, we can provide an easy proof. Now, the order structure of $K(X,Y)$ will be our focus. We derive conditions under which the space has the R.D.P. or is a lattice. Assuming that the positive cone in X is closed, normal, and generating, we first address the latter scenario.

Theorem(4.5): Let Y be a simplex space and let X be a partially ordered Banach space with a closed, normal, and generating cone. If and only if $K(X,Y)$ has the R.D.P, then X has the R.D.P. Here, the implication in one direction will be served by the demonstration that (i) ⟹ (ii) in Proposition (3.16).Once more, we apply Lazar's theorem and Proposition (4.1) to reduce the issue to the space $A_0(K,X^*)$,If all of them belong to $A_0(K,X^*)$,and , all belong to $A_0(K,X^*)$, and

$$\Pi(k) = \{x \in X^* : \sigma(k), \tau(k) \geq x \geq \varphi(k), \psi(k)\},$$

thus $\sigma, \tau \geq \varphi, \psi$,and$\pi \in A_0(K,X^*)$^*) will all be satisfied by any continuous affine selection π of Π.

Now, we have to demonstrate that Π satisfies every requirement of Lazar's theorem. Since X_+^* is closed, it is evident that $\Pi(k)$is non-empty and closed (since X^* is a lattice). Let us now assume that U is open in X and that $\Pi(k_0) \cap U \neq \emptyset$, meaning that $\in U$ has

$$\sigma(k_0), \tau(k_0) \geq x \geq \varphi(k_0), \psi(k_0).$$

Let : $y(k) = \Big(x + \big(\sigma(k) - \sigma(k_0)\big) \wedge \big(\tau(k) - \tau(k_0)\big)\Big) \vee \varphi(k) \vee \psi(k)$.

It is obvious.t $\sigma(k), \tau(k) \geq y(k)\varphi(k), \psi(k)$. It follows that y is a continuous function of k since the positive cone in $X_+^*$is normal and generating, allowing the lattice operations and σ, τ, φ,and $\psi$to be continuous. Therefore, if k is in N, then $y(k) \in U$, and there exists

a neighborhood N of $k_0$in K. Since $y(k)$unquestionably also belongs to $\Pi(k)$,the proof is complete and Π is

lower semi continuous. Next, we examine the requirements for $K(X, Y)$ to be a lattice. First, we demonstrate.

Lemma (4.6): Let Y be a lattice with an approximate unit norm. Y has a supremum for each sparingly closed and bounded subset C, and the set

$$C' = \{\sup(A) : A \subseteq C\},$$

is relatively compactThis lemma has been established in [17] using an order unit for Y. The lattice operations on Y are pointwise operations on the set of extreme points, $\partial_e K$, of the set according to Edwards' theorem.

$$K = \left\{ f_j \in Y^* : f_j \geq 0, \sum_j \|f_j\| \leq 1 \right\},$$

and consequently upon its closure. Y is therefore a closed sublattice of $C(\overline{\partial_e K})$. $\sup(C)$

will exist in $C(\overline{\partial_e K})$if $C \subseteq Y$ This is located in Y since it is formed in [17] as a limit point

of finite suprema of elements of C. The second portion is immediately derived from the order-unit case result.

Corollary (4.7): Assume Y is a simplex space and X is a partially ordered Banach space with closed, normal, and generating circles. The following are therefore equivalent.

(i) A lattice is $K(X, Y)$.

(ii) Y is a lattice, and X possesses the R.D.P.

(i) ⟹ (ii). The proof for this is identical to that of the implication (a) ⟹ (b) in Proposition (3.16).

(ii) ⟹ (i). This is proved in [17], using our Lemma (4.7) instead of [17].

Using our Lemma (4.7) rather than [17], this is demonstrated in [17]. We examine a specific form of $K(X,Y)$.According to [3], if and only if X is base-normed, then this space is order-unit-normed(the proofs of both of which hold in this instance). We can extend this conclusion, unlike the situation where we were working with bounded operators.

Theorem (4.8): Suppose Y is a simplex space and X is a partially ordered Banach space with a dosed cone. If and only if X is base-normed, then $K(X,Y)$has approximate-order-unit average. .$K(X,Y)_+$ is 1-normal, as stated in Proposition (4.3). The open unit ball of $K(X,Y)$ is directed, as theorem (4.4) is demonstrated via a demonstration remarkably similar to this one.$K(X,Y)$ will therefore have the appropriate form.

`On the other hand, consider that $K(X,Y)$has a norm of approximate order unit. Then, by Proposition (4.3), X is$(1+\varepsilon)$--generated for any $\varepsilon > 0$, and $K(X,Y)^*$ is base normed. Demonstrating that the norm is additive on the non-negative of X is sufficient.$p(x) : T \longmapsto f_j(Tx)$ is a positive linear functional on $K(X,Y)$ of norm ‖x‖ if $f_j \in Y_+^*$,$\sum_j \|f_j\| = 1$, and $x \in X_+$.Given that $K(X,Y)^*$ is base-normed, we

$$\|x_1\| + \|x_2\| = \|p(x_1)\| + \|p(x_2)\| = \|p(x_1)\| + \|p(x_2)\| = \|p(x_1 + x_2)\| = \|x_1 + x_2\|$$

whenever $x_1, x_2 \in X_+$.

5.Result:

1. In Corollary (4.7) we have already been able to prove that the two $K(X,Y)$ and R.D.P are equivalent.

$L(X,Y')$ was created in a positive way and $S' \geq T, 0$.

2. that $Y' \subseteq Y$ and that$y \geq 0 \Longrightarrow Py \geq y$ for any suitable linear map P from Y into Y'.The same is true for $L(X,Y')$ if $L(X,Y)$ is positively generated whenever X_+ is normal. T can also be thought of as an element of $L(X,Y)$ if $T \in L(X,Y')$.For $S \in L(X,Y)$, there exists $S \geq T, 0.S' = P \circ S$ if $S' \in L(X,Y')$.Furthermore, $S'x =$

$P(Sx) \geq P(Tx) \geq Tx$ and $S'x = P(Sx) \geq P(0) = 0$ are both true $if\ x \geq 0$. As a result, $L(X, Y')$ is positively generated and $S' \geq T, 0$.

Conclusion:

Reynolds ([18]) states that if $X^*$is base-normed and has a weak*-compact base, then X is order-unit-normed. According to Ellis ([3]). X is base-normed if and only if X^* has order-unit norm. (Ng ([19])).

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